

电磁学

静电场 { 真空 (导体) Maxwell. 偶极子
 介质 本构方程. $\vec{D}$ 边界 球极化 (椭球) 唯一性
 导体 镜像 本构.
 场能. 相互作用能 电势 虚功
 (解 $\nabla\varphi$ { 格林
 分解
 积分)

静磁场 { 真空 Maxwell. $\vec{B}$ 霍尔 磁偶极子 超导体.
 介质. 本构. $\vec{H}$ $\vec{B}$ $\vec{M}$ 边界.
 磁路? 唯一性.

电磁 { Maxwell. 法拉第 $\vec{E} \sim \frac{\partial \vec{B}}{\partial t}$
 磁 $\nabla \times \vec{B} \sim \mu_0 \vec{j} + \epsilon_0 \frac{\partial \vec{E}}{\partial t}$
 能流. 能量密度

电磁波

$$\left. \begin{aligned}
 ① \nabla \cdot \vec{E}(\vec{r}, t) &= \frac{1}{\epsilon_0} \rho(\vec{r}, t) \\
 ② \nabla \cdot \vec{B}(\vec{r}, t) &= 0 \\
 ③ \nabla \times \vec{E}(\vec{r}, t) &= -\frac{\partial \vec{B}(\vec{r}, t)}{\partial t} \\
 ④ \nabla \times \vec{B}(\vec{r}, t) &= \mu_0 \left[\vec{j}(\vec{r}, t) + \epsilon_0 \frac{\partial \vec{E}(\vec{r}, t)}{\partial t} \right]
 \end{aligned} \right\}$$

(i)

$$\nabla(fg) = f\nabla g + g\nabla f.$$

$$(ii) \nabla(\mathbf{A} \cdot \mathbf{B}) = \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A}) + (\mathbf{A} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{A}.$$

two for divergences:

$$(iii) \nabla \cdot (f\mathbf{A}) = f(\nabla \cdot \mathbf{A}) + \mathbf{A} \cdot (\nabla f),$$

$$(iv) \nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B}),$$

and two for curls:

$$(v) \nabla \times (f\mathbf{A}) = f(\nabla \times \mathbf{A}) - \mathbf{A} \times (\nabla f),$$

$$(vi) \nabla \times (\mathbf{A} \times \mathbf{B}) = (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B} + \mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A}).$$

$$\nabla(\vec{r} \cdot \vec{p}) = \vec{p}$$

$$\nabla \times (f(\vec{r})\vec{r}) = 0 \quad r \neq 0$$

$$(\vec{A} \cdot \nabla)\vec{r} = \vec{A}$$

一. 静电学.

$$\S 1.1 \text{ 真空: } \left\{ \begin{array}{l} \nabla \cdot \vec{E} = \rho / \epsilon_0 \\ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = 0 \Rightarrow \varphi \end{array} \right.$$

$$\begin{aligned} \text{(叠加原理)} \quad \varphi(\vec{r}) &= \sum_i \frac{q_i}{4\pi\epsilon_0 r_i} = \int \frac{\rho(\vec{r}')}{4\pi\epsilon_0 r} dv' & \vec{r} &= \vec{r} - \vec{r}' \\ \vec{E}(\vec{r}) &= -\nabla\varphi = \sum_i \frac{q_i}{4\pi\epsilon_0 r^2} \hat{r} = \int \frac{\rho(\vec{r}')}{4\pi\epsilon_0 r^2} \hat{r} dv' & \nabla \frac{1}{r} &= -\frac{\vec{r}}{r^3} \end{aligned}$$



偶极子.

$$\begin{aligned} \text{远场: } \varphi &= \varphi_+ + \varphi_- = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{|\vec{r} - \frac{\vec{d}}{2}|} - \frac{1}{|\vec{r} + \frac{\vec{d}}{2}|} \right) \\ &\sim -\frac{q}{4\pi\epsilon_0} \nabla \frac{1}{r} \cdot \vec{d} \\ \vec{p} &= q\vec{d} = \frac{q}{4\pi\epsilon_0} \frac{\vec{d} \cdot \vec{r}}{r^3} = \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot \vec{r}}{r^3} \end{aligned}$$

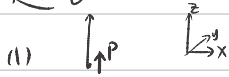
$$\nabla \frac{1}{r} = -\frac{1}{r^3} \nabla r = -\frac{\hat{r}}{r^2} = -\frac{\vec{r}}{r^3}$$

$$\begin{aligned} \vec{E} &= -\nabla\varphi = -\frac{1}{4\pi\epsilon_0} \nabla \frac{\vec{p} \cdot \vec{r}}{r^3} = \frac{1}{4\pi\epsilon_0} \left[\vec{p} \cdot \vec{r} \nabla \frac{1}{r^3} + \frac{1}{r^3} \nabla(\vec{p} \cdot \vec{r}) \right] \\ &= \frac{1}{4\pi\epsilon_0} \left[3 \frac{(\vec{p} \cdot \vec{r})\vec{r}}{r^5} - \frac{\vec{p}}{r^3} \right] \end{aligned}$$

$$\text{偶极子在外场能量 } \mathcal{W} = q\varphi_+ - q\varphi_- = q\varphi(\vec{r} + \frac{\vec{d}}{2}) - q\varphi(\vec{r} - \frac{\vec{d}}{2}) = q \cdot \nabla\varphi \cdot \vec{d} = \vec{p} \cdot \vec{E}$$

(计算相互作用能须乘 $\frac{1}{2}$ 再叠加.)

题6



$$(1) \quad \vec{r} = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{\sqrt{x^2+y^2+z^2}} - \frac{1}{\sqrt{x^2+y^2+z^2}} \right) \quad \text{or} \quad \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$$

$$\vec{E} = -\nabla\varphi \Rightarrow \begin{cases} E_x = -\frac{\partial\varphi}{\partial x} = \frac{q}{4\pi\epsilon_0} \left(\frac{x}{\sqrt{x^2+y^2+z^2}} - \frac{x}{\sqrt{x^2+y^2+z^2}} \right) \\ E_y = -\frac{\partial\varphi}{\partial y} = \frac{q}{4\pi\epsilon_0} \left(\frac{y}{\sqrt{x^2+y^2+z^2}} - \frac{y}{\sqrt{x^2+y^2+z^2}} \right) \\ E_z = -\frac{\partial\varphi}{\partial z} = \frac{q}{4\pi\epsilon_0} \left(\frac{z}{\sqrt{x^2+y^2+z^2}} - \frac{z}{\sqrt{x^2+y^2+z^2}} \right) \end{cases}$$

$$\vec{E} = \frac{qd}{4\pi\epsilon_0} \begin{cases} E_x = \frac{xz}{r^3} \\ E_y = \frac{yz}{r^3} \\ E_z = \left(3\frac{z^2}{r^3} - \frac{1}{r^3} \right) \end{cases}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \left(3\frac{(\vec{r}\vec{r})}{r^5} - \frac{\vec{r}}{r^3} \right)$$

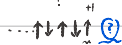
(2)



$$\mathcal{W} = \sum_j -\frac{1}{2} \vec{p}_j \cdot \vec{E}_j \quad \vec{p}_j = q_j \hat{z} \quad \vec{p}_j \cdot \vec{E}_j = q_j p \left(E_j \right)_z = -q_j p \frac{q_j p}{|j-j|^3} = \frac{q_j q_j p^2}{|j-j|^3}$$

$$= -\frac{p^2}{2} \sum_j \frac{q_j q_j}{|j-j|^3}$$

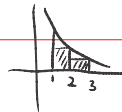
收敛



$$\Delta W = \frac{p^2}{\epsilon_0} \sum_j \frac{(-1)^j}{j^3}$$

$$= \frac{p^2}{\epsilon_0} \left(-\frac{1}{1^3} + \frac{1}{2^3} - \frac{1}{3^3} + \dots \right)$$

$$\therefore \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{4^3} + \dots < \int_1^{\infty} \frac{1}{x^3} dx = \frac{1}{2} \quad \therefore \sum_n = \frac{1}{2^3} - \frac{1}{3^3} + \frac{1}{4^3} + \dots + \frac{(-1)^n}{n^3} < \frac{1}{2^3} + \dots + \frac{1}{n^3} \leq \frac{1}{2}$$



$$\therefore \Delta W < 0$$

则以后放置时 $\Delta W > 0$

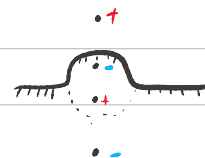
显然的。

§1.2 导体.

① $\vec{j} = \sigma \vec{E}$ (有源, 非静平衡)

② 等势屏蔽, 平衡时 $\vec{E} = 0$ $\varphi = \text{const.} \Rightarrow E_{\perp} = 0$ 高斯 $\Rightarrow E_{\perp} = \frac{\sigma}{\epsilon_0}$

③ 镜像: 凌等势 (电荷守恒) (唯一性) $\Rightarrow$ 电荷 $\Rightarrow$ 势.



求出 $\vec{E}$, 即用高斯求出 σ .

题2. RBP 288

测量: $\left. \begin{array}{l} \vdots \\ \vdots \end{array} \right\} \text{电容}$

$$\frac{dE}{dx} = -\left(\frac{1}{R_1} + \frac{1}{R_2}\right)E$$

Gauss + 几何

$$\Delta(E \cdot dS) = 0$$

R 取极值时保证曲面所在面垂直于平面.

§ 1.3 介质

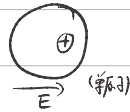


$$\vec{D} = \vec{P} + \vec{E} \quad \left\{ \begin{array}{l} \vec{P} = \frac{\epsilon - 1}{4\pi} \vec{E} \Rightarrow \nabla \cdot \vec{P} = -\rho_p; \text{边界 } \sigma_p = \hat{n} \cdot (\vec{P}_1 - \vec{P}_2) \\ \vec{D} = \epsilon \vec{E} + \vec{P} \Rightarrow \nabla \cdot \vec{D} = \rho - \rho_p = \rho \quad \text{边界 } \sigma_f = \hat{n} \cdot (\vec{D}_1 - \vec{D}_2) \end{array} \right.$$

线性均匀极化

取向极化

位移极化



$$P = 4\pi\epsilon_0 E_0 R^3$$

$\hat{n}$ 由1到2

$$dA = \frac{ds}{r^2} \propto d\cos\theta$$

$$= P \frac{2}{\eta} \int_0^{\eta} \int_0^{\cos\theta} d\cos\theta = P (\coth\eta - \frac{1}{\eta})$$

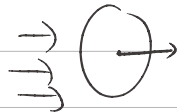
$$<\eta = \frac{P}{kT}>$$

$$\text{def: } \vec{P} = \chi \epsilon_0 \vec{E} \Rightarrow \vec{D} = \epsilon_0 \vec{E} + \vec{P} = \epsilon_0 (1 + \chi) \vec{E} = \epsilon_r \epsilon_0 \vec{E}$$

(本构关系, 即介电性质) $<\epsilon_r = 1 + \chi>$

求解问题: $\vec{P} = (\epsilon_r - 1) \epsilon_0 \vec{E}$ 设 $\sigma_p = \vec{P} \cdot \hat{n}$ 求 E' 代入 $E = E_0 + E'$ 求出 $E \Rightarrow \sigma_p$
(假设 $\vec{E}$ 与 $\vec{E}_0$ 同向, $\vec{E} = \vec{E}_0 + \vec{E}'$, 若不是, 难)

例



$$E_{\text{in}} = \frac{3}{\epsilon_r + 2} E_0 = E_0 + \frac{1 - \epsilon_r}{\epsilon_r + 2} E_0 \quad P = (\epsilon_r - 1) \epsilon_0 E = 3 \epsilon_0 \frac{\epsilon_r - 1}{\epsilon_r + 2} E_0$$

$$\sigma_p = 3 \epsilon_0 \frac{\epsilon_r - 1}{\epsilon_r + 2} E_0 \cos\theta \quad \phi_{\text{out}} = -E_0 r \cos\theta + E_0 \frac{\epsilon_r - 1}{\epsilon_r + 2} \frac{R_0^3}{r^2} \cos\theta$$

$$P = 4\pi\epsilon_0 E_0 \frac{\epsilon_r - 1}{\epsilon_r + 2} R_0^3$$

§ 1.4 能量

$$\frac{1}{2} \epsilon E \quad \frac{1}{2} E \cdot P$$

静电: $\frac{1}{2} \epsilon \vec{E} \cdot \vec{D}$ 场能密度 + 极化能 = 系统能.

也可用 $\frac{1}{2} \int \rho \varphi dv$

$$\int_V (\nabla \cdot \vec{D}) dv = \int_V (\nabla \cdot \vec{D} + \nabla \cdot \vec{P}) dv = \int_V \vec{D} \cdot \vec{E} dv + \int_V \rho \varphi dv$$

全空间积分

相互作用能: $\frac{1}{2} \int (\rho_1 + \rho_2)(\varphi_1 + \varphi_2) dv = \int \rho_1 \varphi_2 dv = \int \rho_2 \varphi_1 dv$

$$= \frac{1}{2} \int \rho_1 \varphi_1 dv + \frac{1}{2} \int \rho_2 \varphi_2 dv + \frac{1}{2} \int (\rho_1 \varphi_2 + \rho_2 \varphi_1) dv$$

(1) (2)

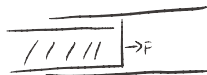
电容: $C = \frac{Q}{U}$

平行板  $C = \frac{\sigma S}{E d} = \frac{\epsilon_0 S}{d}$

串  $\Rightarrow C = \frac{Q}{U_1 + U_2} \Rightarrow \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

并  $\Rightarrow C = \frac{Q_1 + Q_2}{U} \Rightarrow C = C_1 + C_2$

题1



自由荷

$$1. (1) \sigma_1 = \frac{U_0}{d} \cdot \epsilon_r \epsilon_0$$

$$\sigma_2 = \frac{U_0}{d} \epsilon_0 \quad Q = \sigma_1 x \cdot b + \sigma_2 (a-x)b = \frac{U_0}{d} \epsilon_0 b (\epsilon_r x + a - x)$$

也可串并联

$$\text{体系 } W = \frac{1}{2} Q U_0 = \frac{1}{2} \frac{\epsilon_r b}{d} U_0^2 (\epsilon_r x + a - x)$$

$$\text{虚功: } U_0 dQ = dW + F dx$$

$$\Rightarrow F = \frac{1}{2} \frac{\epsilon_r b}{d} (\epsilon_r - 1) U_0^2$$

$$(2) \text{ 设 } Q, \text{ 并联: } C = C_1 + C_2 = \frac{\epsilon_0 \epsilon_r x b}{d} + \frac{\epsilon_0 (a-x)b}{d}$$

$$W = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} Q^2 \frac{d}{\epsilon_0 b} \frac{1}{\epsilon_r x + a - x}$$

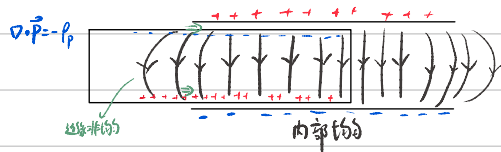
$$F dx + dW = 0$$

$$F = \frac{1}{2} Q^2 \frac{d}{\epsilon_0 b} \frac{\epsilon_r - 1}{(\epsilon_r x + a - x)^2}$$

$$\text{代入 } Q = C U_0 = \frac{\epsilon_r b}{d} (\epsilon_r x + a - x) U_0$$

$$\Rightarrow F = \frac{1}{2} \frac{\epsilon_r b}{d} (\epsilon_r - 1) U_0^2$$

(3) 利用 (4)



$$\text{偶极子法 } F = \int \vec{p} \cdot \nabla \vec{E} \, dv$$

$$= \int (\epsilon_r - 1) \epsilon_0 (\vec{E} \cdot \nabla) \vec{E} \, dv$$

$$= (\epsilon_r - 1) \epsilon_0 \int \nabla \cdot (\vec{E} \vec{E}) \, dv \quad (\text{张量运算})$$

$$= \frac{(\epsilon_r - 1) \epsilon_0}{2} \oint dS \cdot \vec{E}^2$$

$$= \frac{(\epsilon_r - 1) \epsilon_0}{2} b \cdot d \left(\frac{U_0}{d} \right)^2 = \frac{(\epsilon_r - 1) \epsilon_0}{2} \frac{U_0^2}{d} b$$

$$\nabla(\vec{A} \cdot \vec{B}) = (\nabla \vec{A}) \cdot \vec{B} + (\vec{A} \cdot \nabla) \vec{B} + (\nabla \vec{B}) \cdot \vec{A} + (\vec{B} \cdot \nabla) \vec{A}$$

$$\Rightarrow \nabla(\vec{E} \cdot \vec{E}) = 2(\nabla \vec{E}) \cdot \vec{E} + 2(\vec{E} \cdot \nabla) \vec{E} \Rightarrow \nabla \cdot \vec{E}^2 = 2(\vec{E} \cdot \nabla) \vec{E}$$

$$\epsilon_r^2 E^2 - \epsilon_r + (\epsilon_r - 1)$$

$$\epsilon_r^2 - 1$$