

Chapter 8

Waves

water waves, sounds, light, radio and
TV broadcasting
ecology problem
nerve impulse
infections or rumor spreading
wave, ripples, tide, surge

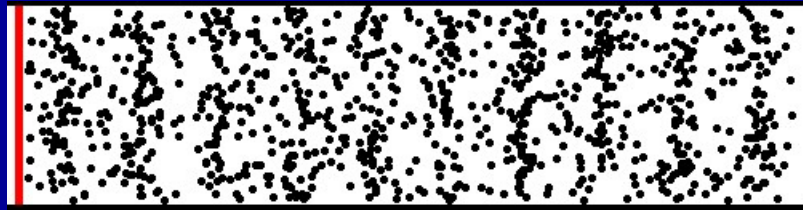
*Wave is propagation of **oscillation** or **field***

8.1 Waves and their classification

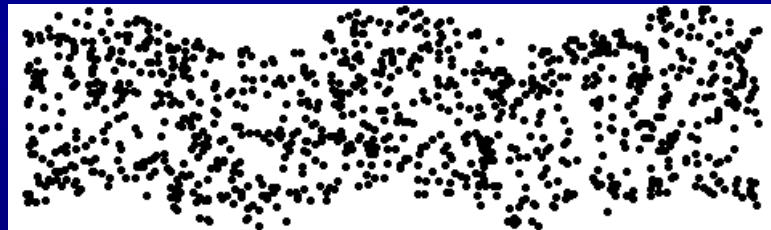
Mechanical waves medium
electromagnetic waves

directions of the displacement and propagation

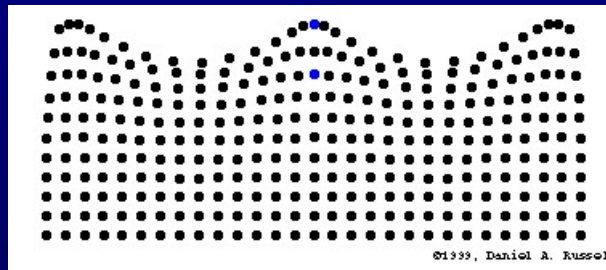
longitudinal



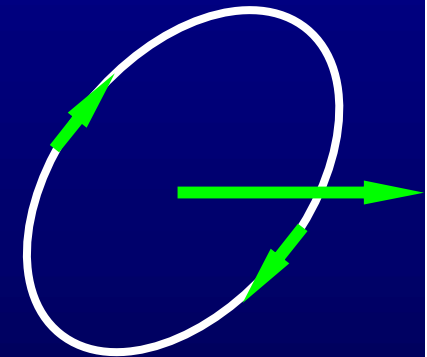
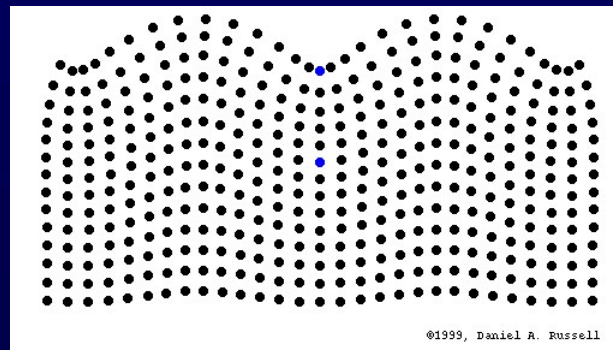
transverse



water wave
mixed



Rayleigh
surface waves
mixed



Dimension

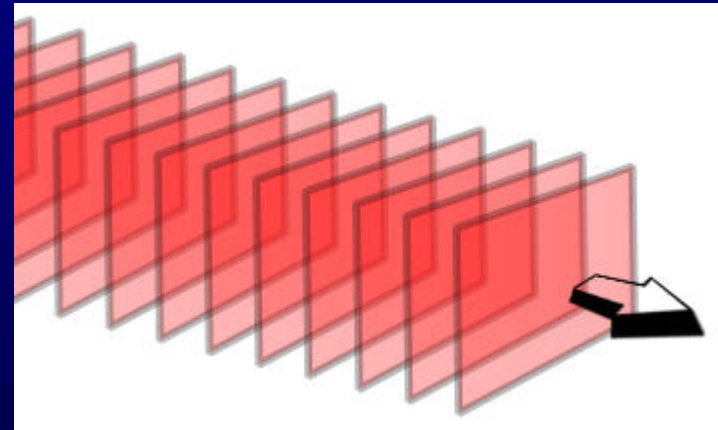
- 1D (along a string)
- 2D (surface wave, ripple)
- 3D (bulk)

Wavefronts

- planar
- spherical (point source)
- cylindrical (linear source)

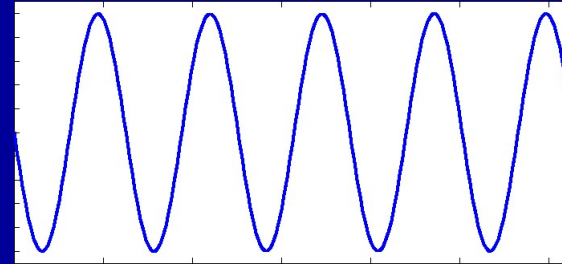
Wavefronts

- planar
- spherical (point source)
- cylindrical (linear source)



Shapes

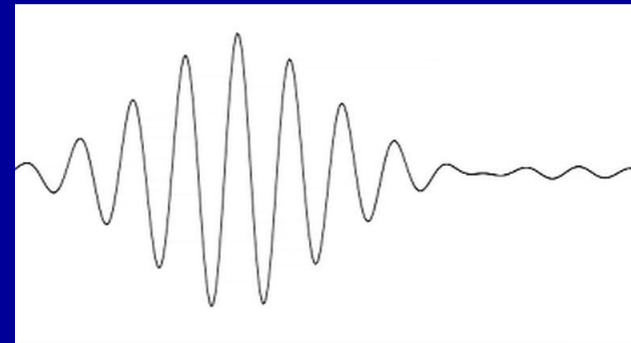
- **wavetrain**



- **pulse**



- **packet**



Wave equation

- linear
- nonlinear waves
(**shock waves or solitary waves**)
- travelling
- standing waves

Wave-particle duality

Example: Seismic waves

Body waves

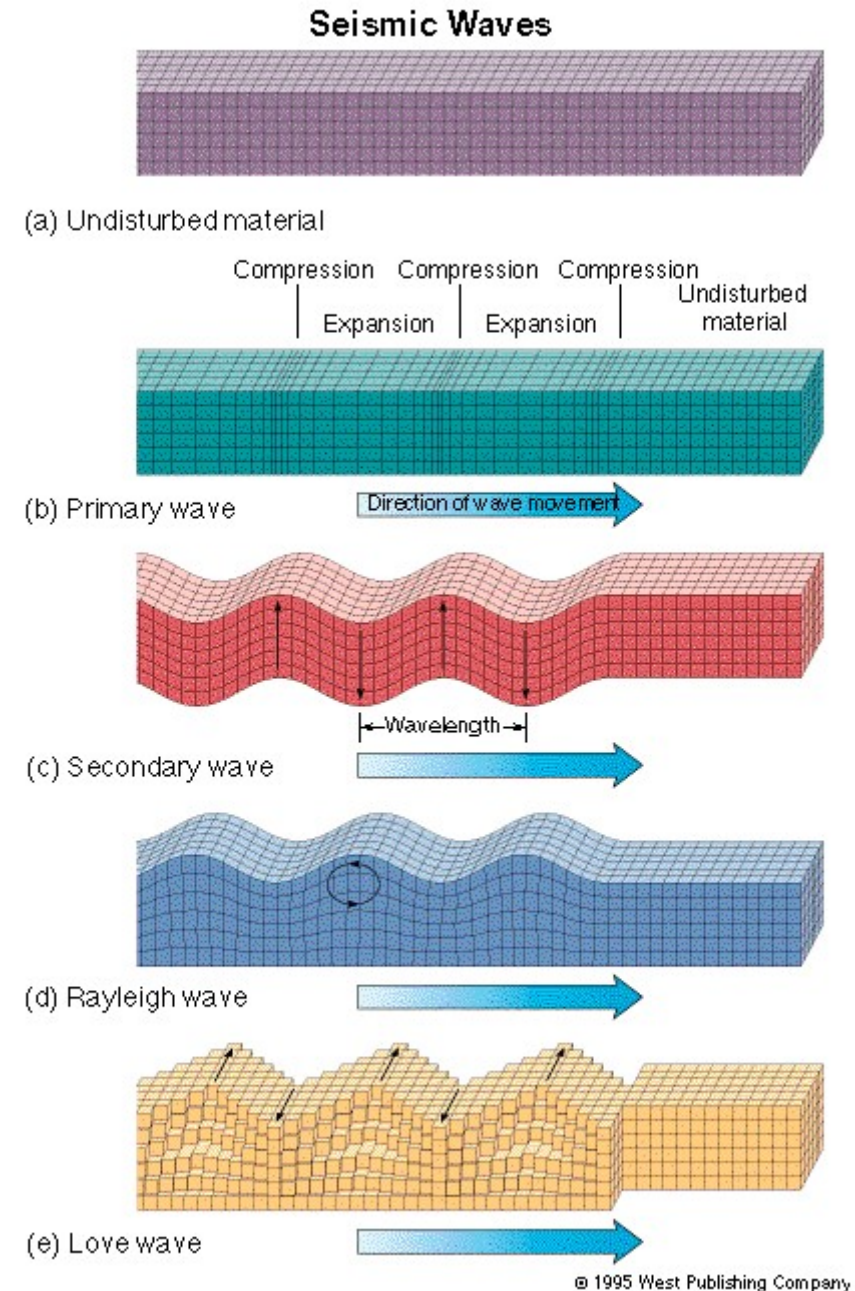
P wave (primary wave)

S wave (shear or secondary wave)

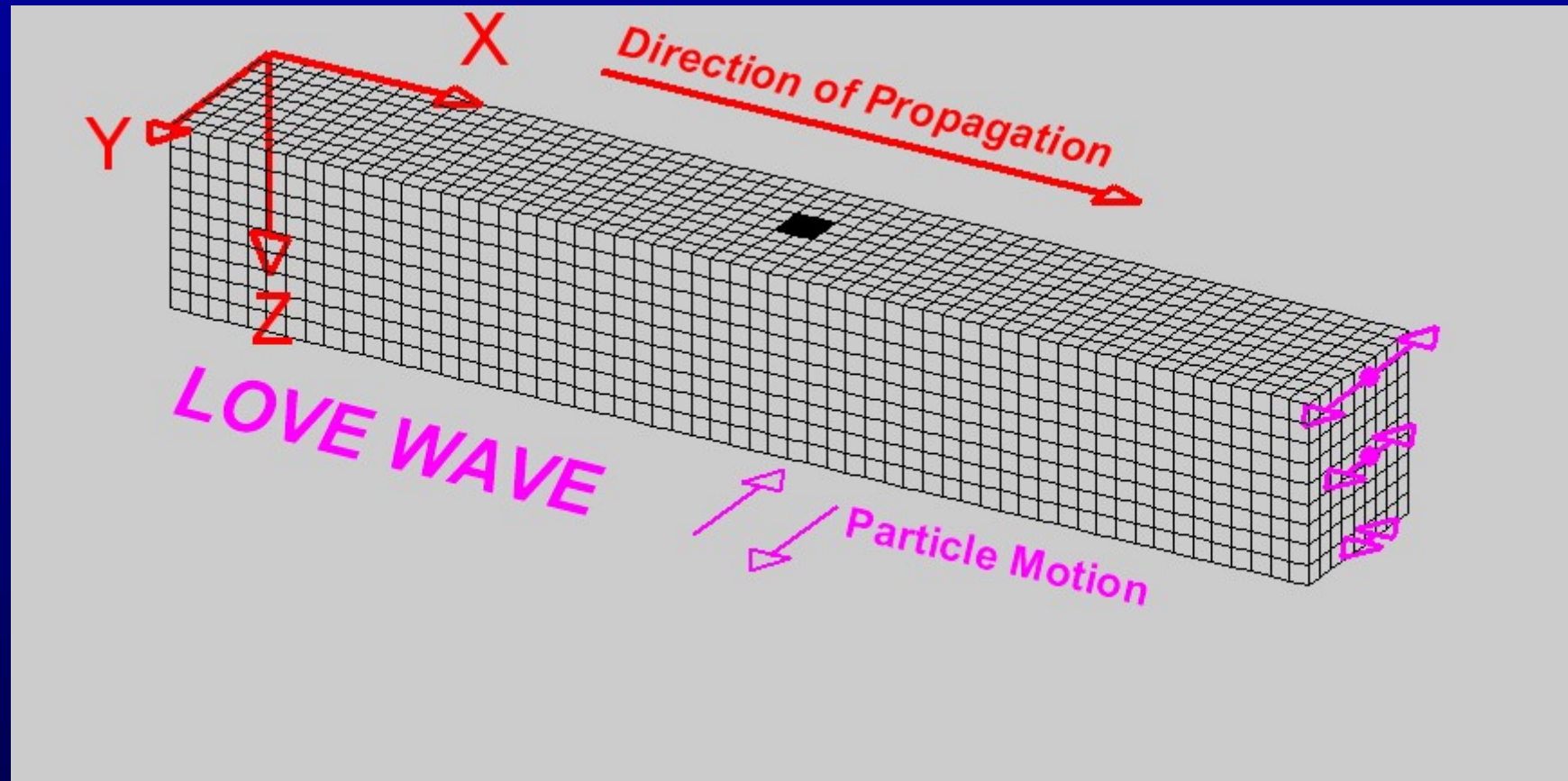
Surface waves

Rayleigh wave

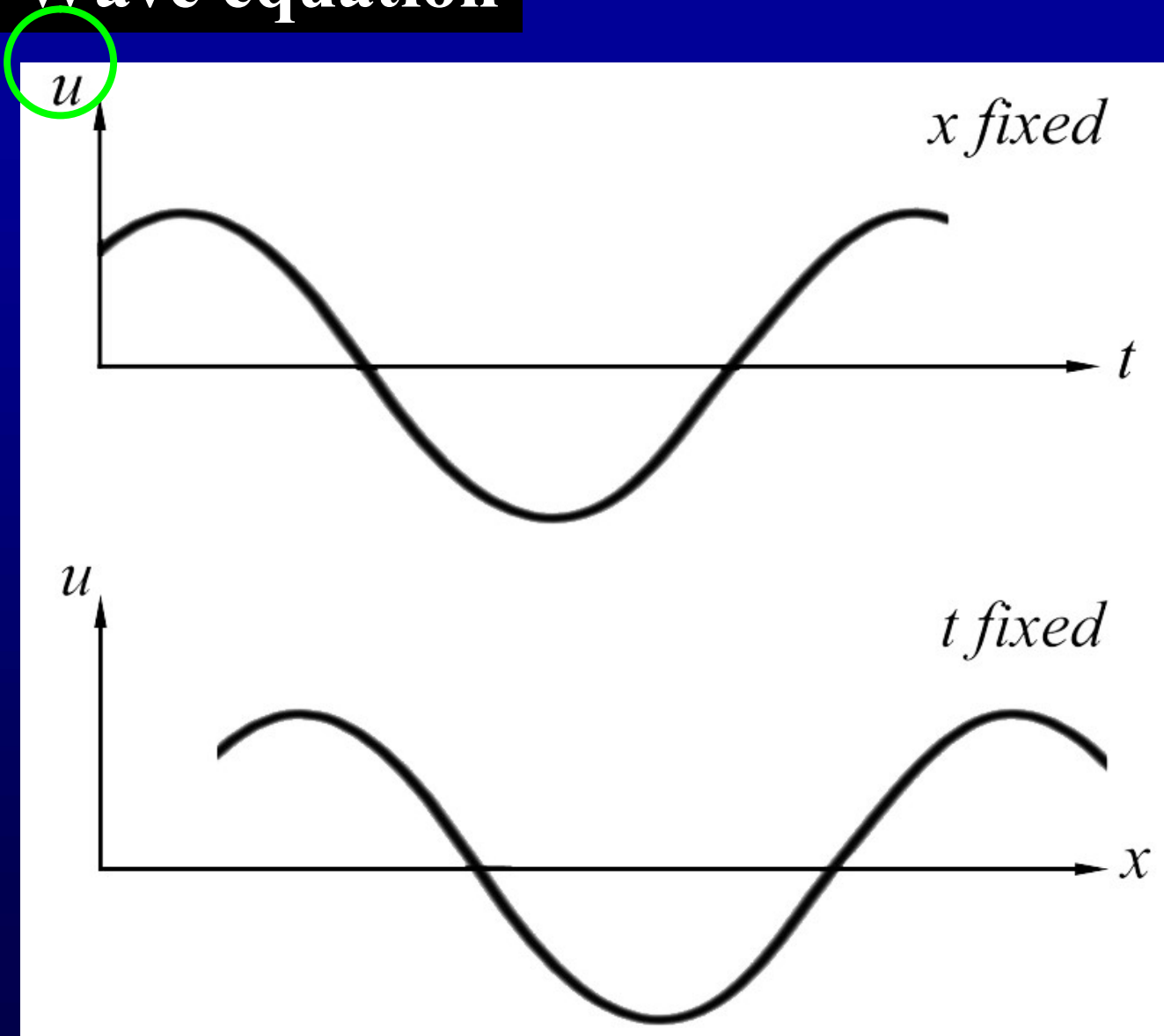
Love wave



Love wave



8.2 Wave equation



$t = 0$ snapshot

$$u(x, 0) = f(x) \quad u(x, t) = ?$$

In S' , moving along with the wave

$$u(x, t) = f(x')$$

For a right-traveling wave

$$x' = x - vt, \quad u(x, t) = f(x - vt)$$

For a left-traveling wave

$$u(x, t) = f(x + vt)$$

$$u(x, t) = f(x - vt)$$

$$u(x, t) = f(x + vt)$$

$$u_t = f'(x \mp vt) \times (\mp v)$$

$$u_x = f'(x \mp vt) \quad u_t = \mp v u_x$$

$$u_t + v u_x = 0 \quad (\text{right traveling})$$

$$u_t - v u_x = 0 \quad (\text{left traveling})$$

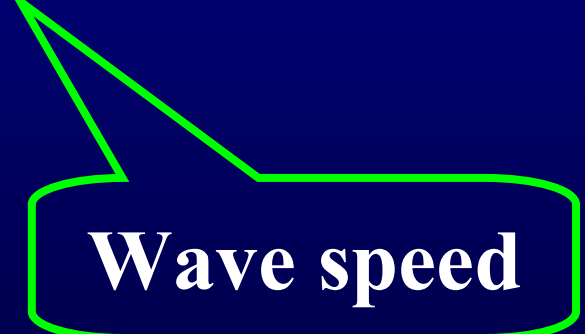
traveling wave equation

$$u_t = \mp v u_x$$

$$u_{tt} = \mp v u_{xt} = \mp v u_{tx} = \mp v (\mp v u_x)_x = v^2 u_{xx}$$

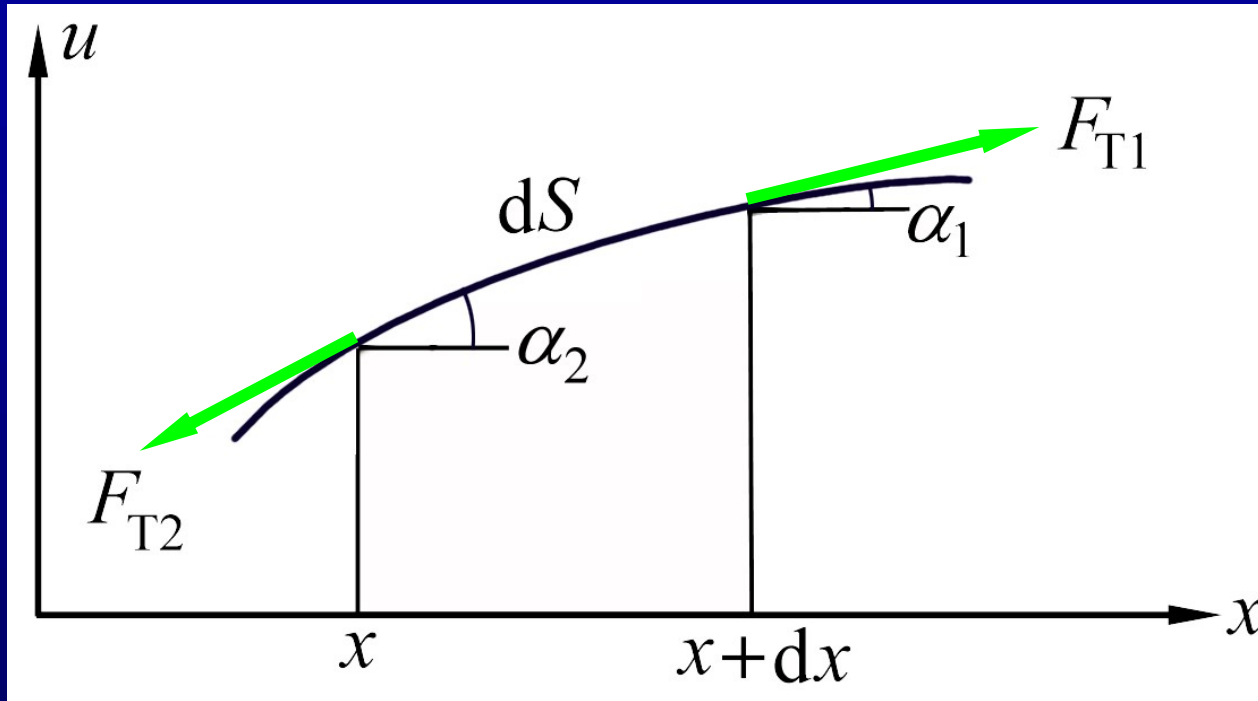
$$u_{tt} - v^2 u_{xx} = 0 \quad \text{or} \quad u_{xx} - \frac{1}{v^2} u_{tt} = 0$$

Wave equation



Wave speed

transverse wave on flexible string



$$F_{T1} \cos \alpha_1 - F_{T2} \cos \alpha_2 = 0$$

$$F_{T1} \sin \alpha_1 - F_{T2} \sin \alpha_2 = (\rho \, ds) \frac{\partial^2 u}{\partial t^2}$$

$$\cos \alpha_1 \approx \cos \alpha_2 \approx 1 \quad \sin \alpha \approx \tan \alpha \approx \frac{\partial u}{\partial x}$$

$$ds = \sqrt{dx^2 + du^2} = dx \sqrt{1 + \left(\frac{\partial u}{\partial x} \right)^2} \approx dx$$

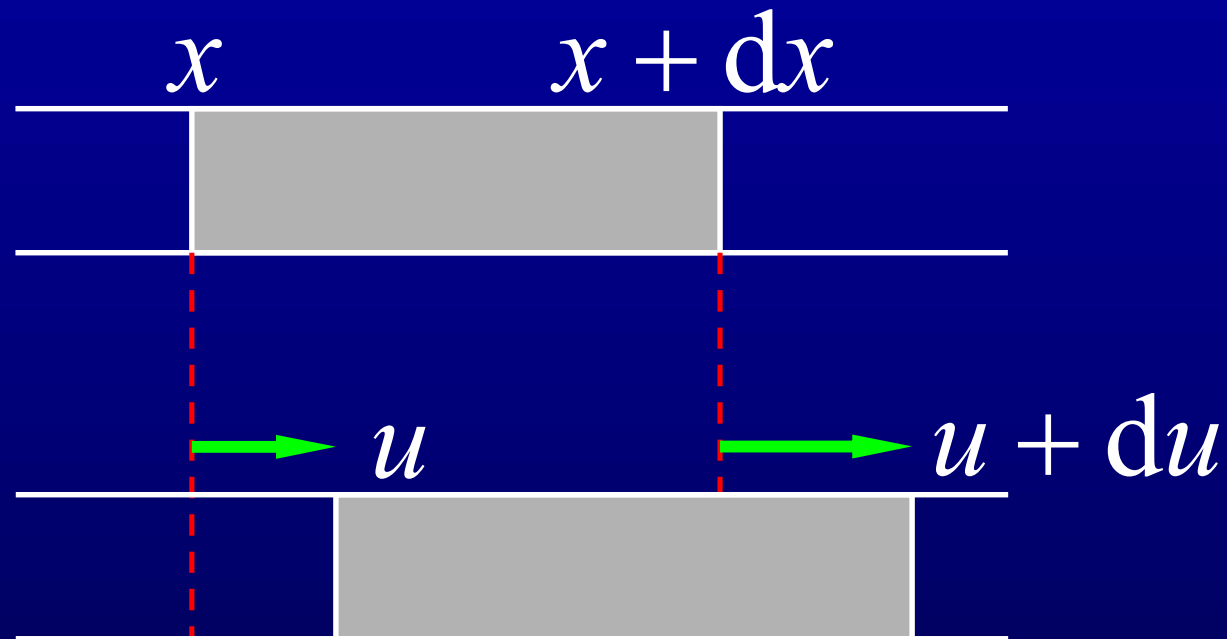
$$F_{T1} = F_{T2} \equiv F_T$$

$$F_T \frac{\partial u}{\partial x} \Big|_{x+dx} - F_T \frac{\partial u}{\partial x} \Big|_x = \rho dx \frac{\partial^2 u}{\partial t^2}$$

$$\frac{\partial u}{\partial x} \Big|_{x+dx} = \frac{\partial u}{\partial x} \Big|_x + \frac{\partial^2 u}{\partial x^2} \Big|_x dx + \dots$$

$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{v^2} \frac{\partial^2 u}{\partial t^2} = 0, \quad v = \sqrt{\frac{F_T}{\rho}}$$

longitudinal wave



Original length L : dx

Stretch ΔL : $du - dx?$ $du?$

$$(\rho_0 dx \cdot S) \frac{\partial^2 u}{\partial t^2} = F$$

Young's modulus

$$\Delta F_T = YS \frac{\Delta L}{L} \quad Y \equiv \frac{L}{S} \left(\frac{\partial F_T}{\partial L} \right)$$

$$F = YS \frac{\partial u}{\partial x} \Big|_{x+dx} - YS \frac{\partial u}{\partial x} \Big|_x = YS \frac{\partial^2 u}{\partial x^2} dx$$

~~$$\rho_0 dx S \frac{\partial^2 u}{\partial t^2} = YS \frac{\partial^2 u}{\partial x^2} dx$$~~

$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} = 0 \quad \left(c = \sqrt{\frac{Y}{\rho_0}} \right)$$

Young's modulus (杨氏模量) also known as the elastic modulus

area of the cross-section

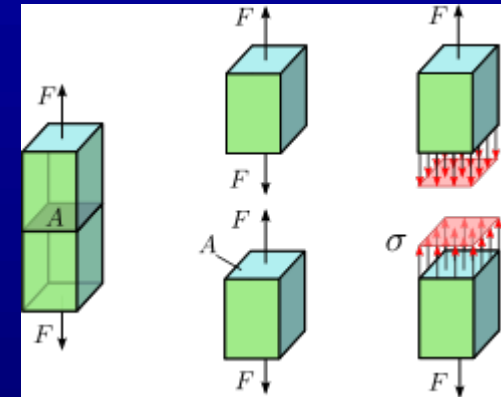
$$F_T = k\Delta L = Y \frac{S}{L} \Delta L$$

total length

$$\sigma \equiv \frac{F_T}{S} = Y \frac{\Delta L}{L} \equiv Y\epsilon.$$

tensile stress 拉应力

tensile strain 拉应变



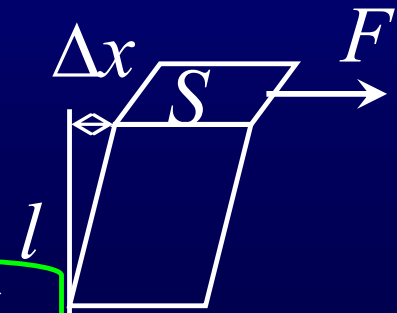
$$Y \equiv \frac{L}{S} \left(\frac{\partial F_T}{\partial L} \right) = \frac{\sigma}{\epsilon}.$$

Shear modulus (切变模量)

$$G = \frac{Fl}{S\Delta x} = \frac{F/S}{\Delta x/l} = \frac{\tau}{\gamma}$$

shear stress 切应力

shear strain 切应变



8.3 Simple harmonic wave and their superposition

$$u_{xx} - \frac{1}{v^2} u_{tt} = 0$$

trial solution to the wave equation

$$u(x, t) = A \cos(kx \mp \omega t - \varphi)$$

$$u_{xx} = -k^2 u, \quad u_{tt} = -\omega^2 u$$

u is the solution only if $\omega/k = v$

amplitude

phase

$$u(x, t) = A \cos(\overbrace{kx \mp \omega t - \varphi})$$

initial phase **at** x : $kx - \varphi$

frequency $\nu = \frac{\omega}{2\pi}$

angular (circular) frequency ω

period $T = \frac{1}{\nu} = \frac{2\pi}{\omega}$

wave vector k , wave number k

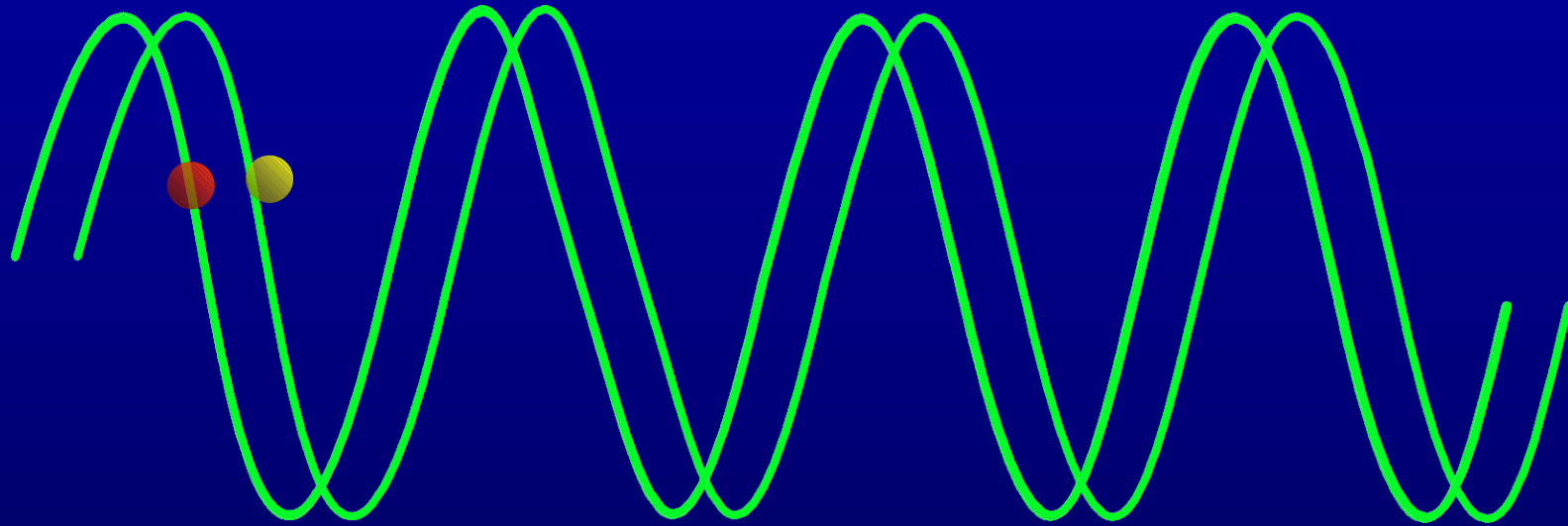
3D: $u(\mathbf{r}, t) = A \cos(\mathbf{k} \cdot \mathbf{r} - \omega t - \varphi)$

$u(\mathbf{r}, t) = \tilde{A} e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)}$, $\tilde{A} = A e^{-i\varphi}$, A is real.

$\text{Re}[\tilde{A} e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)}] = A \cos(\mathbf{k} \cdot \mathbf{r} - \omega t - \varphi).$

Phase speed

(point of same phase)

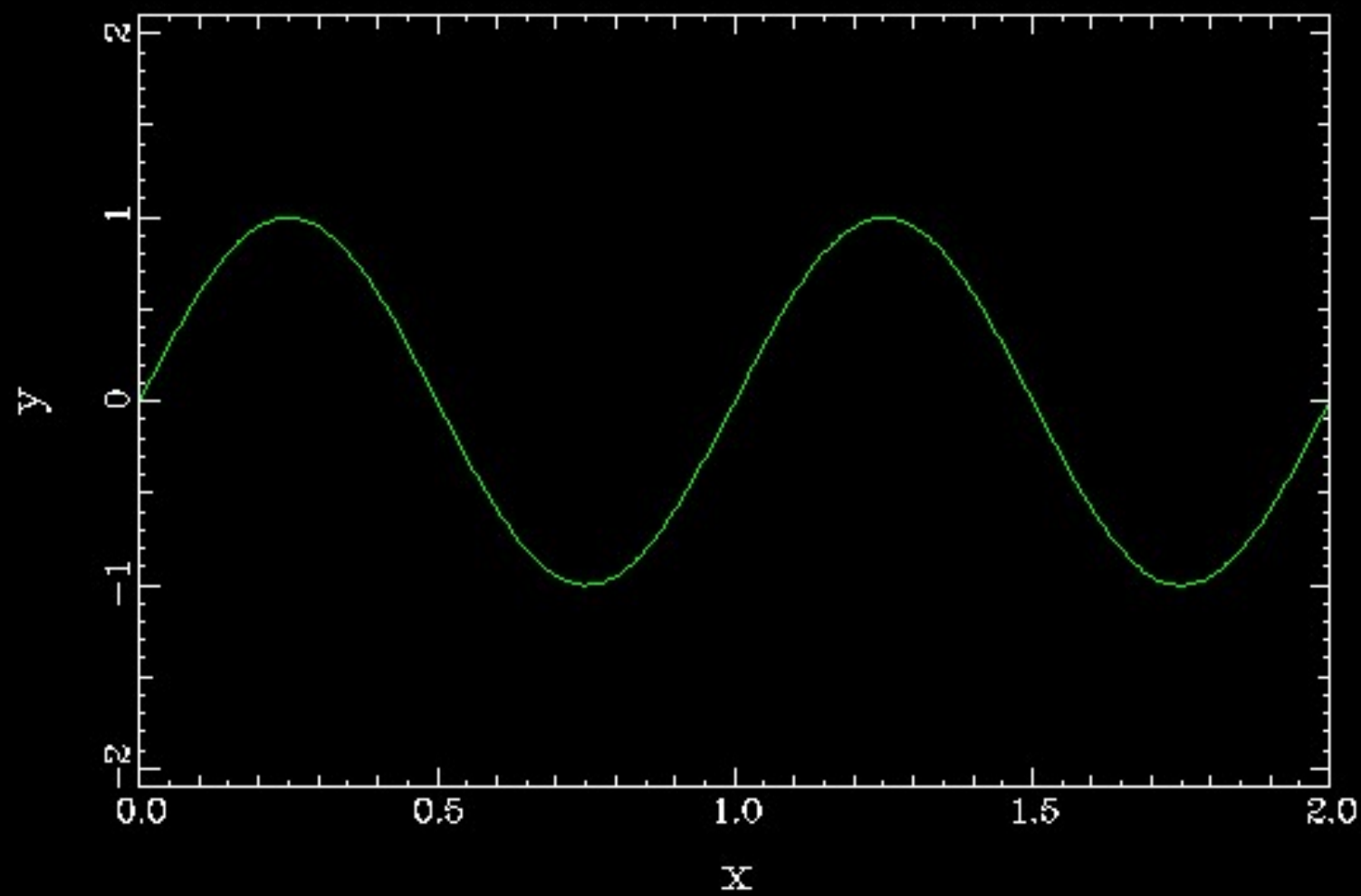


$$\cancel{k(x + \Delta x)} - \cancel{\omega(t + \Delta t)} - \cancel{\varphi} = \cancel{kx} - \cancel{\omega t} - \cancel{\varphi}$$

$$k\Delta x = \omega\Delta t$$

$$v_p = \frac{\Delta x}{\Delta t} = \frac{\omega}{k}$$

One wave



Wavelength λ

$$\lambda = T \cdot v_p = \frac{2\pi}{\omega} \cdot \frac{\omega}{k} = \frac{2\pi}{k}$$

$$[k(x + \underline{n\lambda}) - \omega t - \varphi] - [kx - \omega t - \varphi] = \underline{2n\pi}$$

$$k = \frac{2\pi}{\lambda}$$

$$\omega = \frac{2\pi}{T}$$

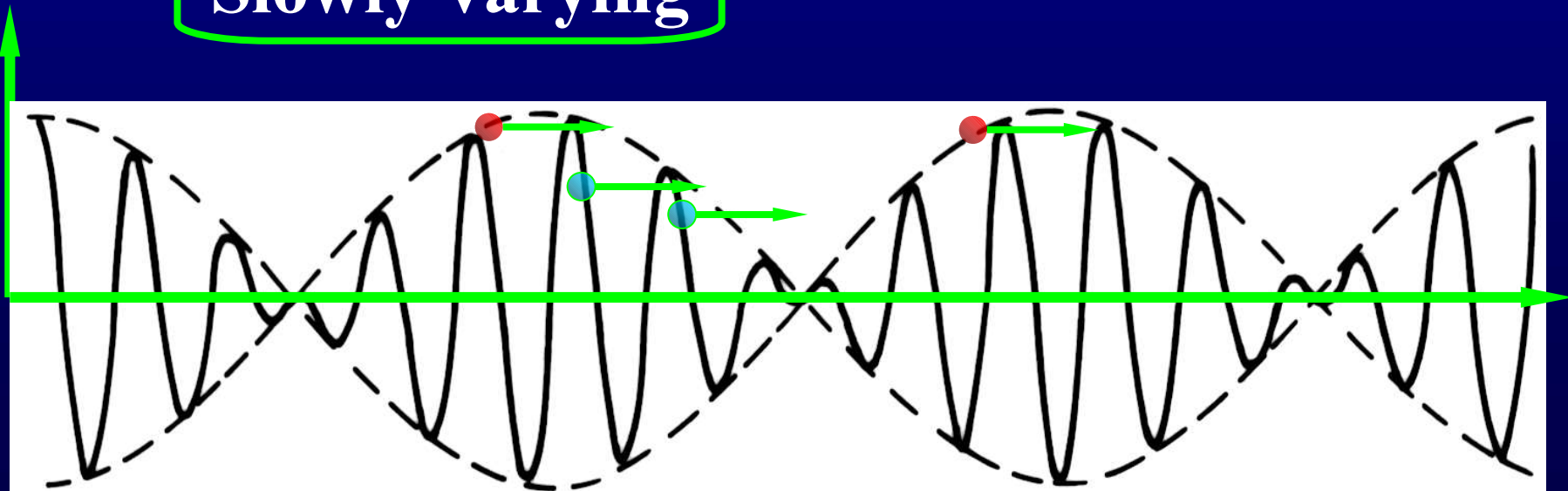
superposition of waves (Fourier analysis)

$$u_1 = A \cos[(k + \Delta k)x - (\omega + \Delta \omega)t]$$

$$u_2 = A \cos[(k - \Delta k)x - (\omega - \Delta \omega)t]$$

$$u_1 + u_2 = 2A \cos(\Delta kx - \Delta \omega t) \cos(kx - \omega t)$$

Slowly-varying



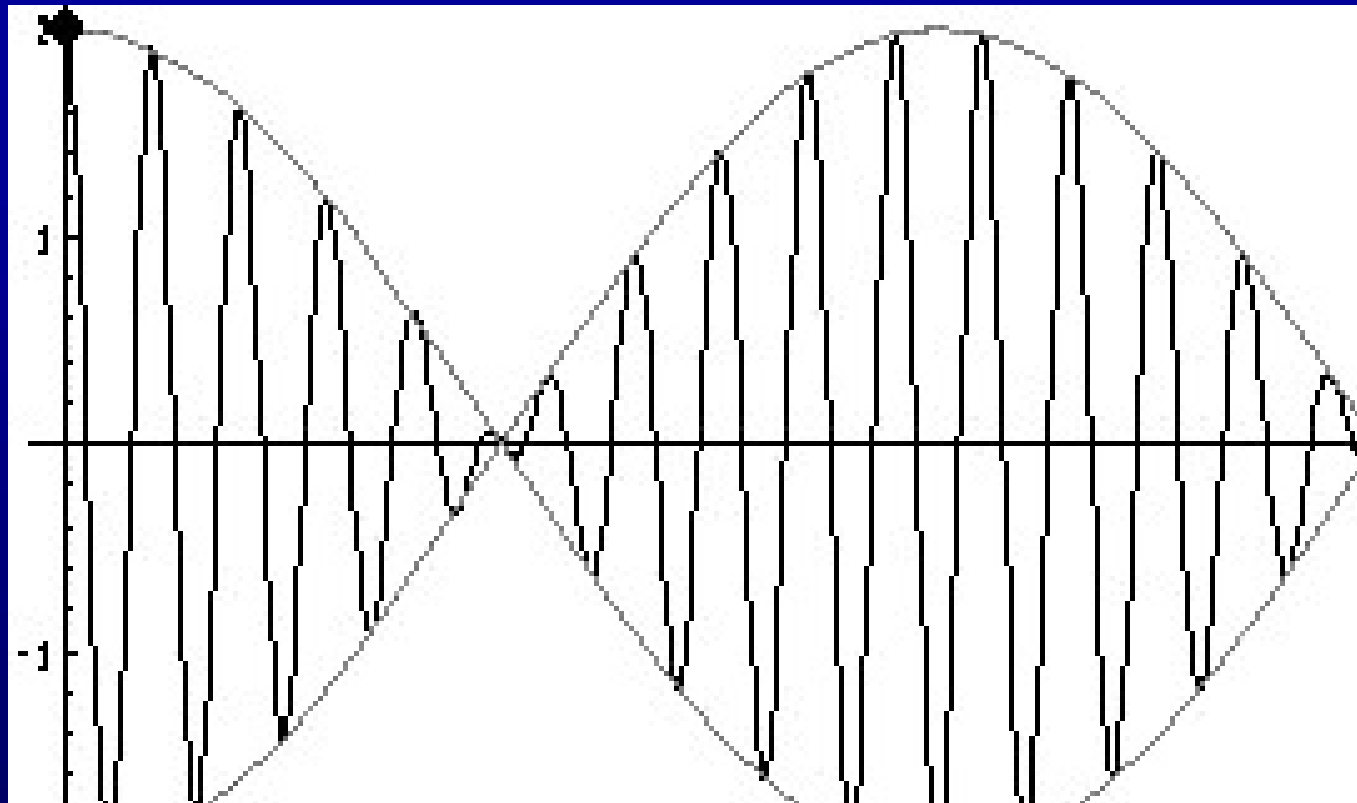
Group speed

the phase speed of the envelope

$$\Delta k \cdot (x + \Delta x) - \Delta \omega \cdot (t + \Delta t) = \Delta k \cdot x - \Delta \omega \cdot t$$

$$\frac{\Delta x}{\Delta t} = \frac{\Delta \omega}{\Delta k}$$

$$v_g = \frac{d\omega}{dk}$$



phase speed vs. group speed

Example: Light or x-ray in solids

The refraction index $n = \frac{c}{v_p} = \frac{kc}{\omega}$

For $n < 1$, $v_p > c$

$$n = 1 - \frac{N q_e^2}{2\varepsilon_0 m \omega^2} \equiv 1 - \frac{a}{\omega^2}$$

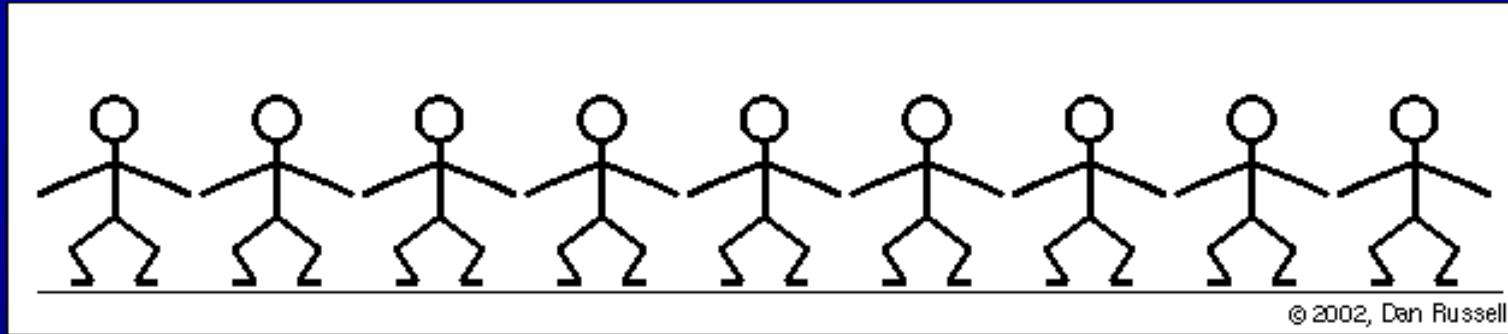
$$v_g = \frac{c}{1 + \frac{a}{\omega^2}} < c$$

$$\frac{kc}{\omega} = 1 - \frac{a}{\omega^2}$$

$$kc = \omega - \frac{a}{\omega}$$

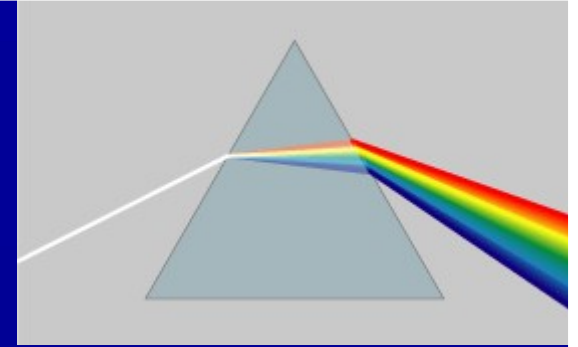
$$c = \frac{d\omega}{dk} + \frac{a}{\omega^2} \frac{d\omega}{dk} = \left(1 + \frac{a}{\omega^2} \right) \frac{d\omega}{dk}$$

$$v_g = \frac{c}{1 + \frac{a}{\omega^2}} < c$$



The phase speed exceeding the speed of light in vacuum does not indicate any superluminal information or energy transfer.

Superluminal group speed



$$v_p = \frac{c}{n(\omega)} = \frac{\omega}{k}$$

$$n(\omega)$$

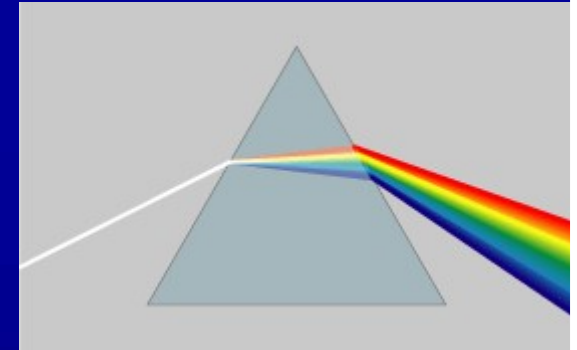
dispersion 色散

$$\omega n(\omega) = ck$$

$$\frac{d\omega}{dk} \left[n(\omega) + \omega \frac{dn(\omega)}{d\omega} \right] = c$$

$$v_g = \frac{d\omega}{dk} = \frac{c}{n(\omega) + \omega \frac{dn(\omega)}{d\omega}} = \frac{c}{n_g}$$

$$v_g = \frac{d\omega}{dk} = \frac{c}{n(\omega) + \omega \frac{dn(\omega)}{d\omega}} = \frac{c}{n_g}$$

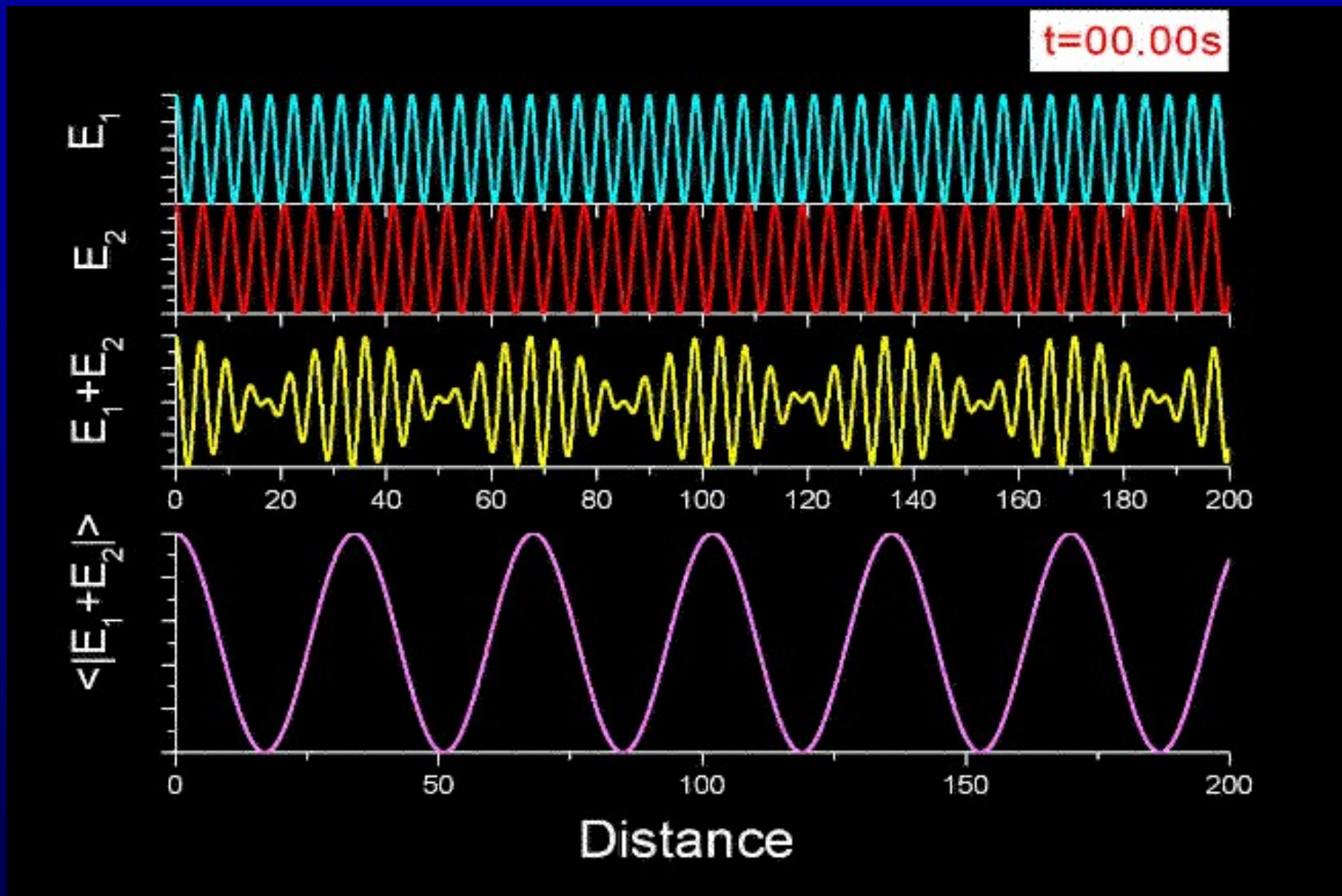


$$\frac{dn(\omega)}{d\omega} > 0 \quad \text{normal dispersion}$$

$$\frac{dn(\omega)}{d\omega} < 0 \quad \text{anomalous dispersion}$$

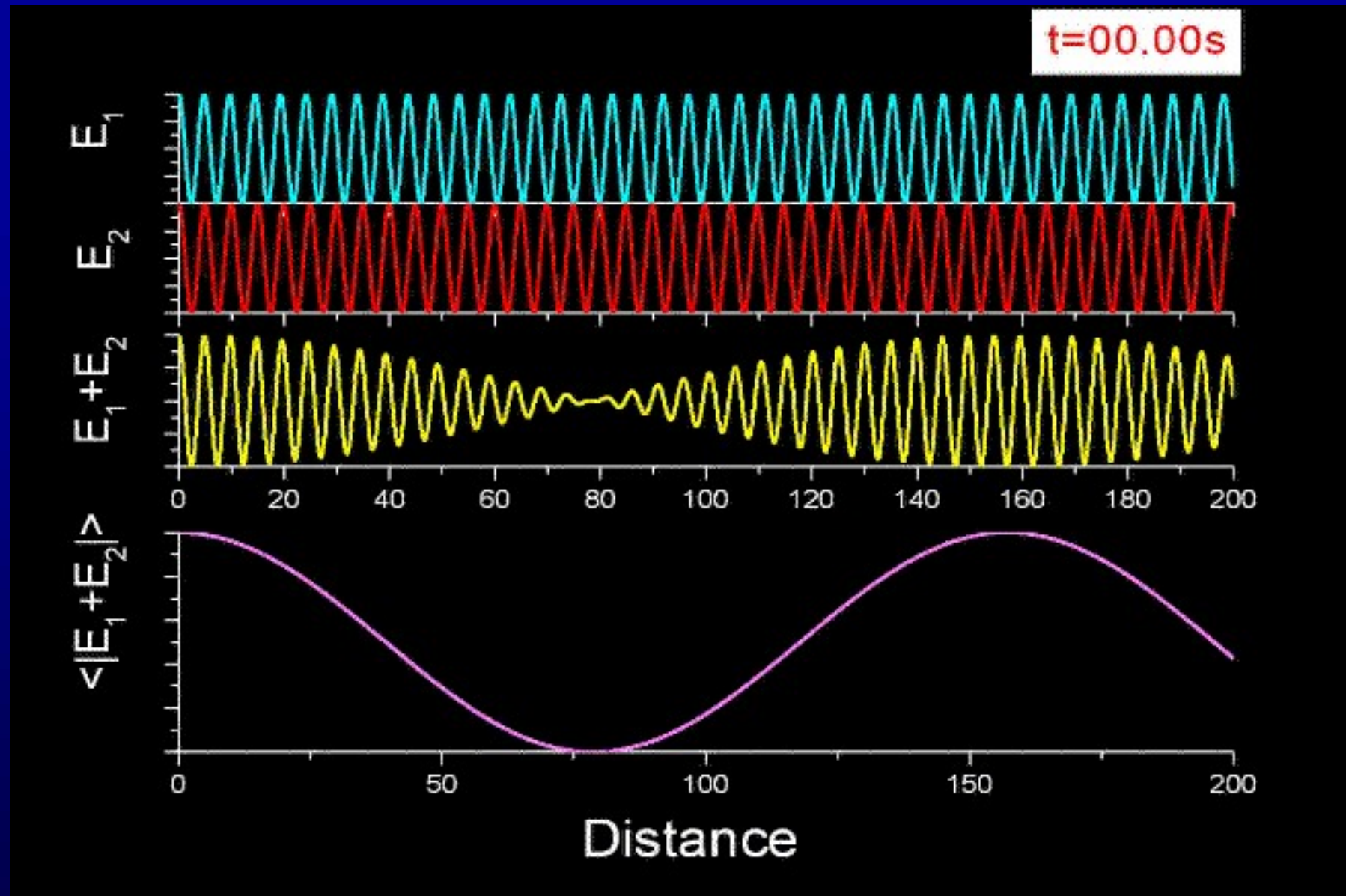
absorbing media
photonic crystals

$$v_g \rightarrow \left\{ \begin{array}{l} > c \text{ superluminal: LJ Wang} \\ & \text{王利军 } Nature, 2000, 406: 277 \\ < 0 \end{array} \right.$$



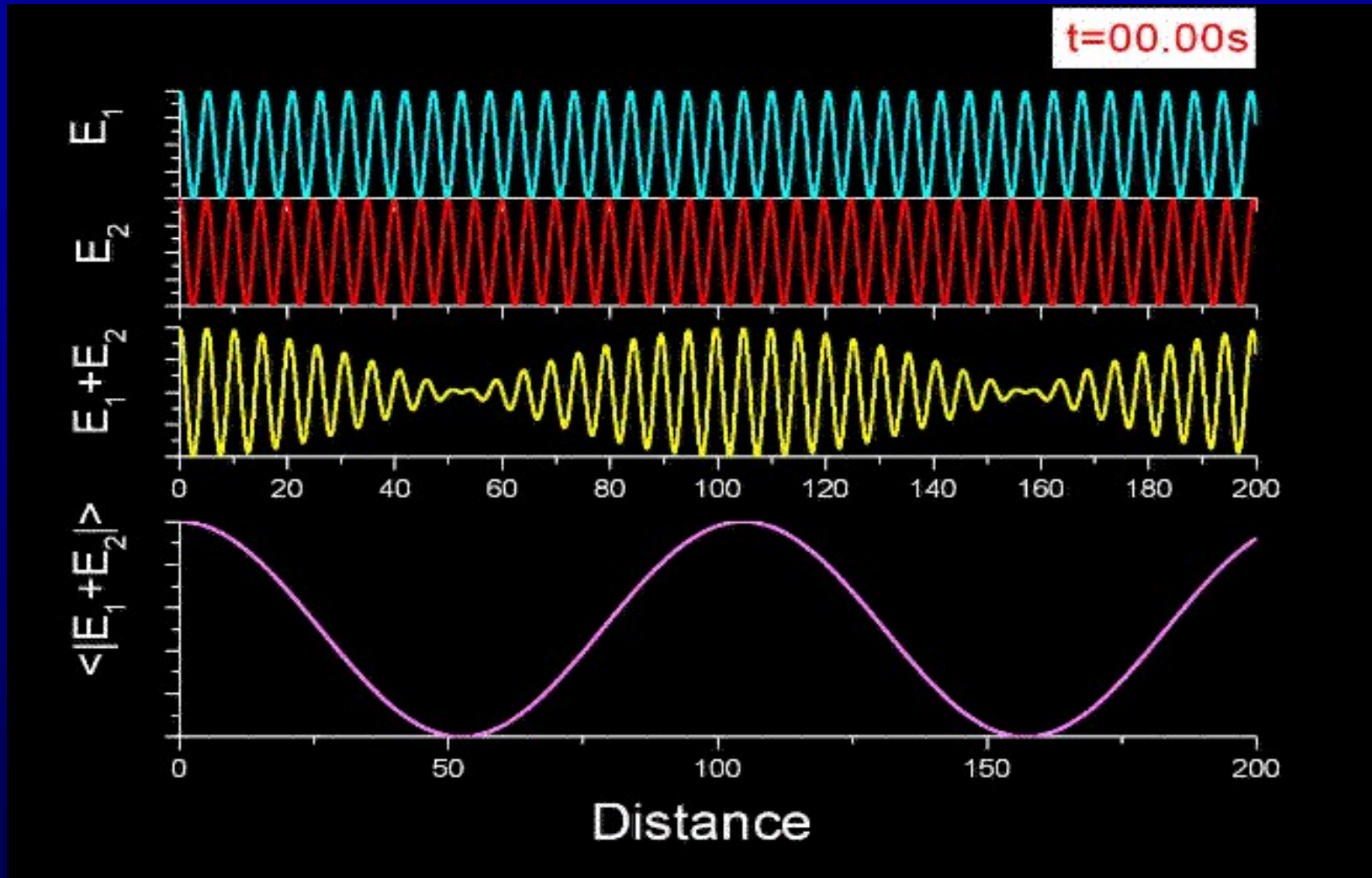
(a) $n(w_1)=1.4$ and $n(w_2)=1.35$; $w_1 > w_2$

$n_g > 1$



(b) $n(w_1)=1.3$ and $n(w_2)=1.4$ $w_1 > w_2$

$1 > n_g > 0$



(c) $n(w_1)=1.2$ and $n(w_2)=1.4$; $w_1 > w_2$

$\mathbf{n_g < 0}$

In the circumstances of anomalous dispersion, the group speed no longer represents the speed at which information or energy propagates.

front velocity

“New information is communicated only when there is an unexpected change, such as a discontinuity, whose arrival time cannot be inferred from the past behavior of the wave.”

RY Chiao and AM Steinberg, Tunneling times and Superluminality, *Progress in Optics* ed. by E. Wolf, XXXVII, p345, 2002.

It is well known that the group velocity v_g of a light pulse can exceed c in an anomalously dispersive medium. As discussed many years ago by Sommerfeld and Brillouin, a group velocity greater than c does not violate causality because it is not the velocity of information transmission. They noted that the "frontal velocity," the velocity at which an infinitely sharp step-function-like disturbance of the light intensity propagates, can serve as a velocity of information transfer. We suggest here an operational definition of the signal velocity and apply it to the recently observed superluminal propagation of a light pulse in a gain medium.

Kuzmich, A. Dogariu, A. Wang, L.J.

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Signal Velocity, Causality, and Quantum Noise in Superluminal Light Pulse Propagation

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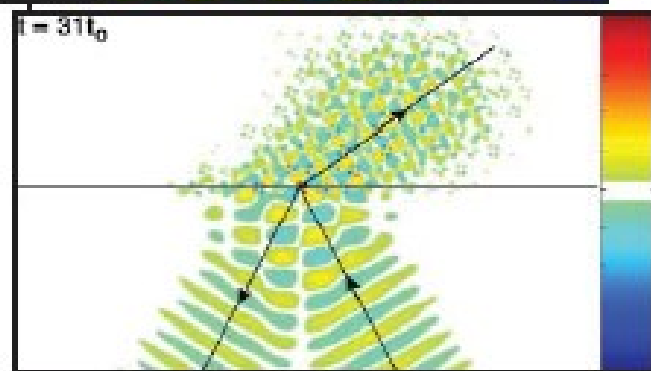
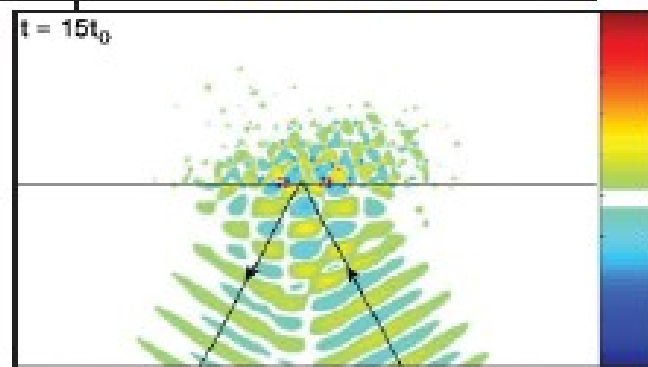
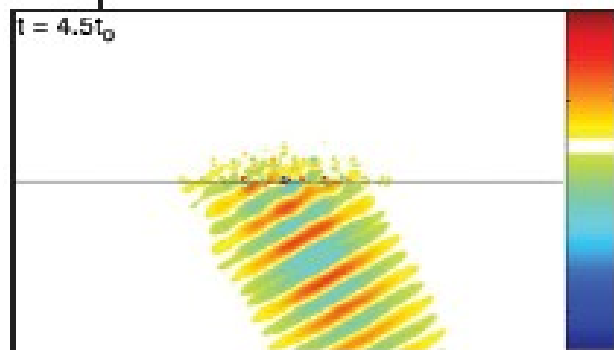
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(Received 3 November 2000)

We consider pulse propagation in a linear anomalously dispersive medium where the group velocity exceeds the speed of light in vacuum (c) or even becomes negative. A signal velocity is defined operationally based on the optical signal-to-noise ratio, and is computed for cases appropriate to the recent experiment where such a negative group velocity was observed. It is found that quantum fluctuations limit the signal velocity to values less than c .



rel, suggesting that current ed with a two- to three-fold in-

PHYSICS

Left-Handed But Not in Left Field

Two years ago, an experiment was devised using a combination of dielectric and conductive elements put together in a specific geometry such that the material would possess negative electric permittivity and magnetic permeability. The result was a demonstration that microwaves entering the material were refracted according to Snell's law, but in the opposite direction to that for "normal" materials, thus confirming an idea first proposed in 1968. Lively discussion ensued, however, about whether these materials really could exist and exhibit these observed properties without breaking the laws of physics. That debate now appears to be put to rest.

Several talks at the March meeting of the American Physical Society and the experimental evidence provided by Parazzoli *et al.* with the simulation work of

Foteinopoulou *et al.* together provide strong evidence supporting the existence of negative index of refraction (also called left-handed) materials. — ISO

Phys. Rev. Lett. 90, 107401; 107402 (2003).

Simulation showing incident, reflected, and refracted waves at the surface of a left-handed material.

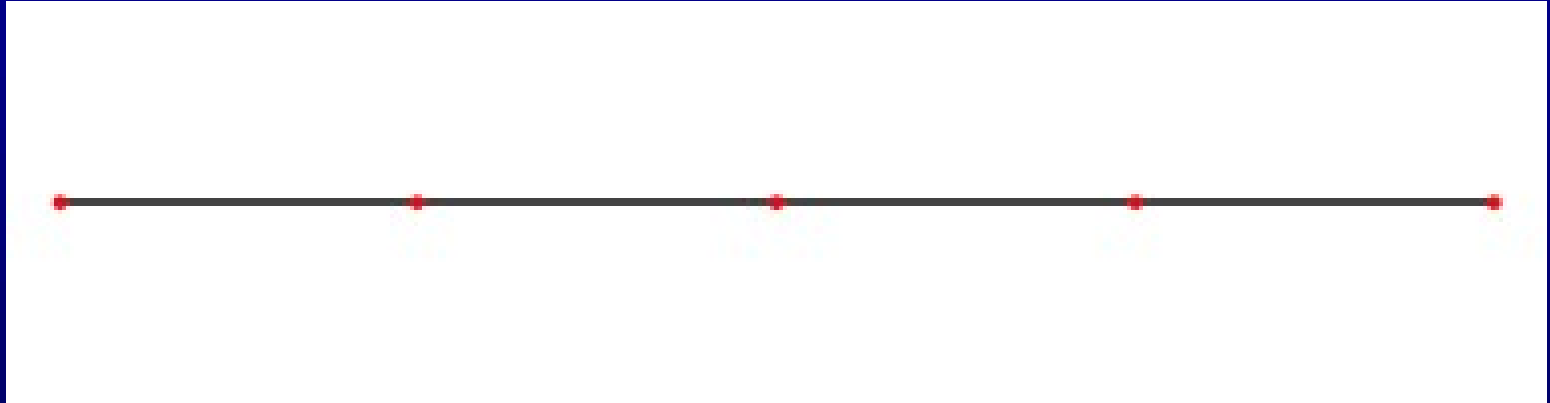
Assignment: 8.1, *8.2, 8.14, 8.15

Standing wave

$$u_1 = A \cos(kx - \omega t - \varphi)$$

$$\begin{aligned} u_2 &= A \cos(kx + \omega t) \\ &= A \cos \left[k \left(\underline{x + \frac{\omega}{k} t} \right) \right] \end{aligned}$$

$$u_1 + u_2 = 2A \cos \left(kx - \frac{\varphi}{2} \right) \cos \left(\omega t + \frac{\varphi}{2} \right)$$



$$(u_1 + u_2)\big|_{x=0} = 0 \longrightarrow \cos\left(-\frac{\varphi}{2}\right) = 0$$

$$\varphi = \pi$$

$$(u_1 + u_2)\big|_{x=L} = 0 \longrightarrow \cos\left(kL - \frac{\varphi}{2}\right) = 0$$

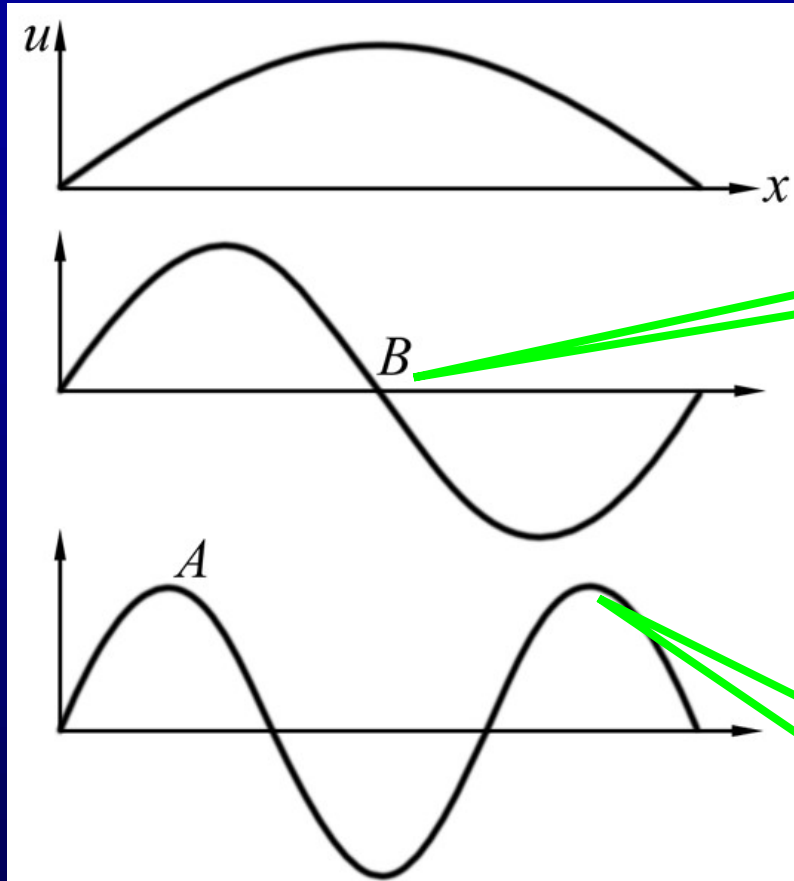
$$kL = n\pi$$

or

$$k = \frac{2\pi}{\lambda} \quad \lambda = 2L/n$$

$$L = n \frac{\lambda}{2}$$

$$L = n \frac{\lambda}{2}$$



node 波节

$$u = 0 = \frac{\partial^2 u}{\partial x^2}$$

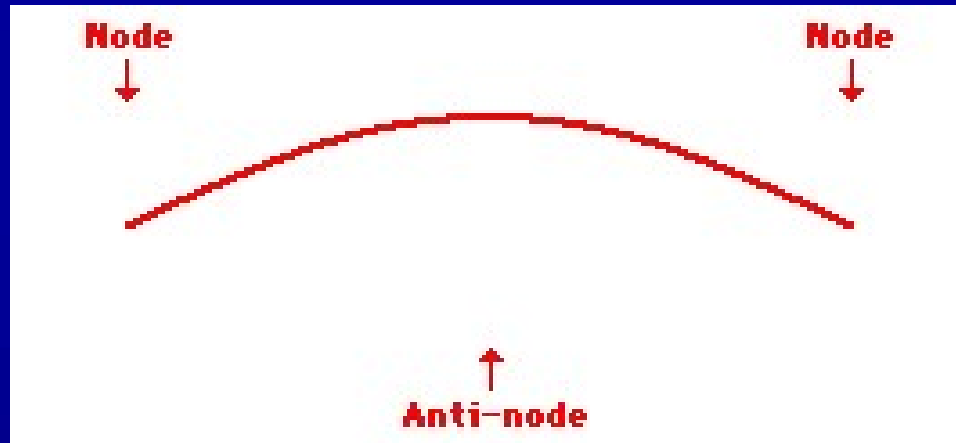
Antinode 波腹

$$f = \frac{\omega}{2\pi} = \frac{kv}{2\pi} = n \frac{v}{2L}$$

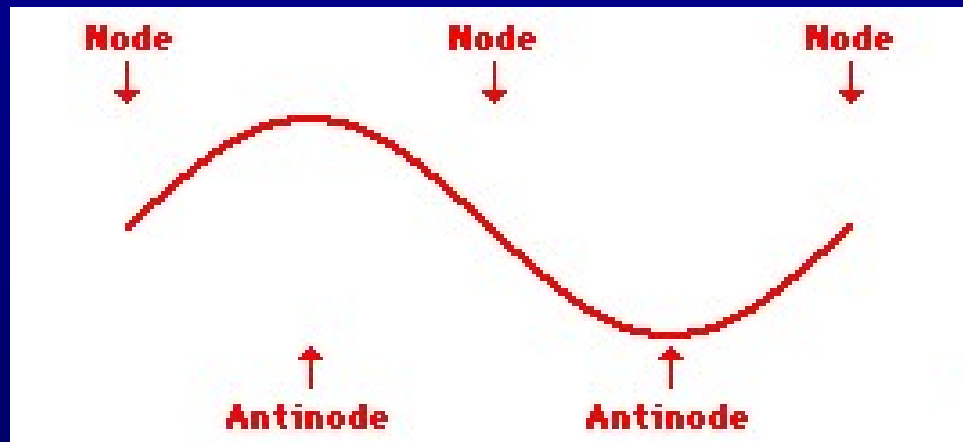
$$f_n \equiv n f_1 \equiv n \frac{v}{2L}$$

harmonics

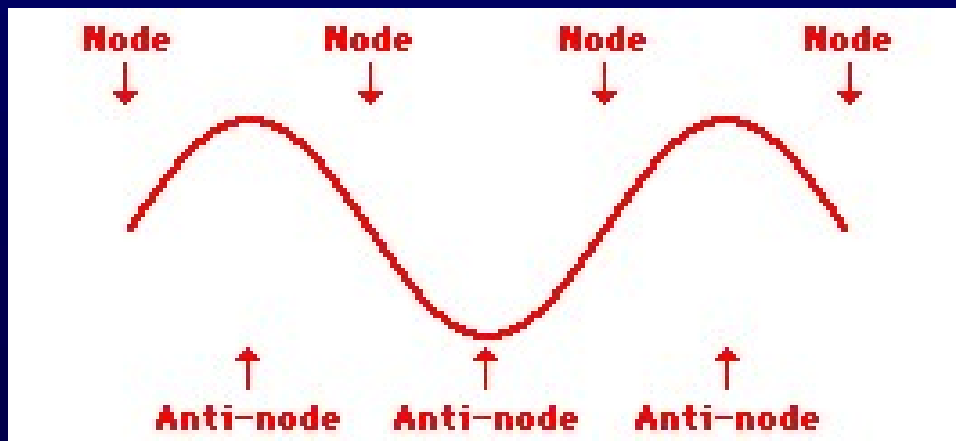
fundamental



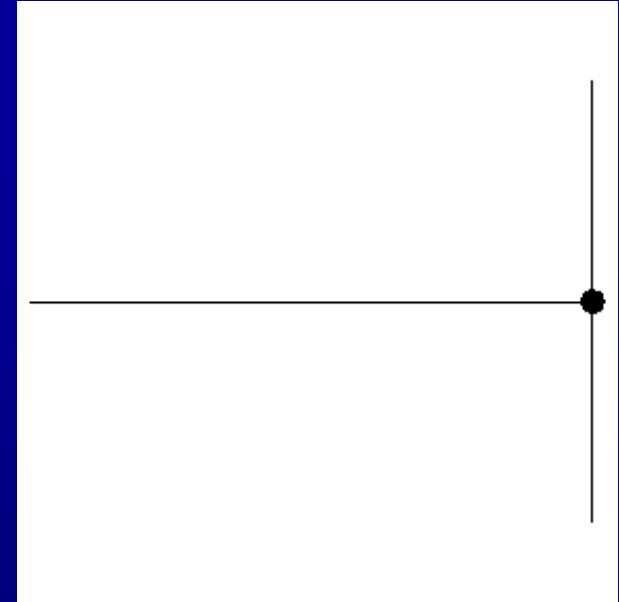
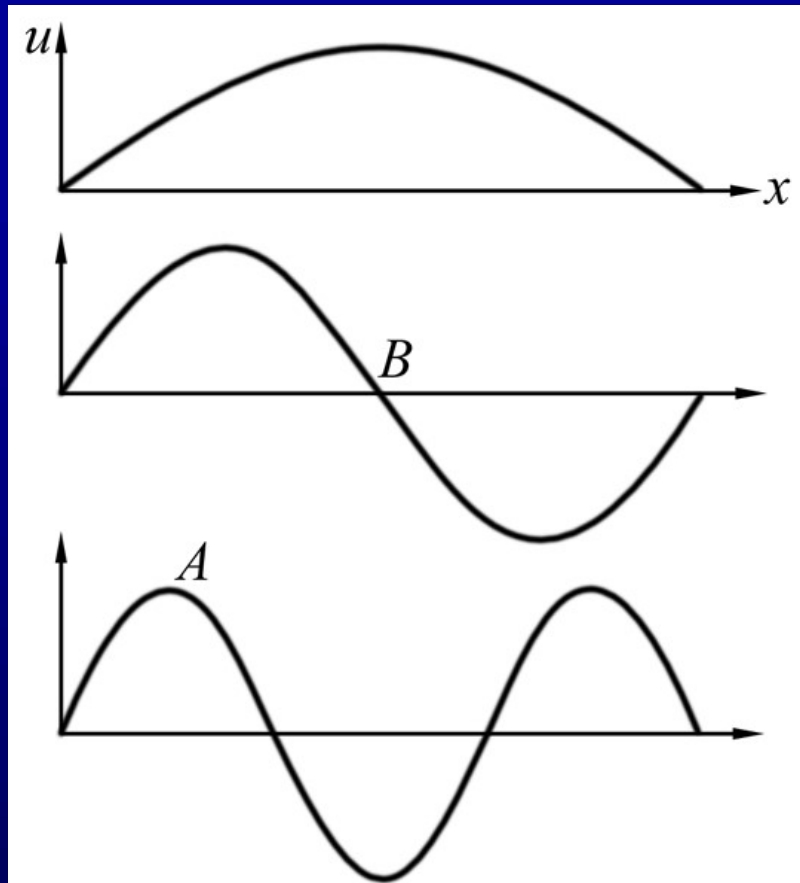
fundamental
first harmonics
一次谐波



second harmonics
first overtone
二次谐波



third harmonics
second overtone
三次谐波

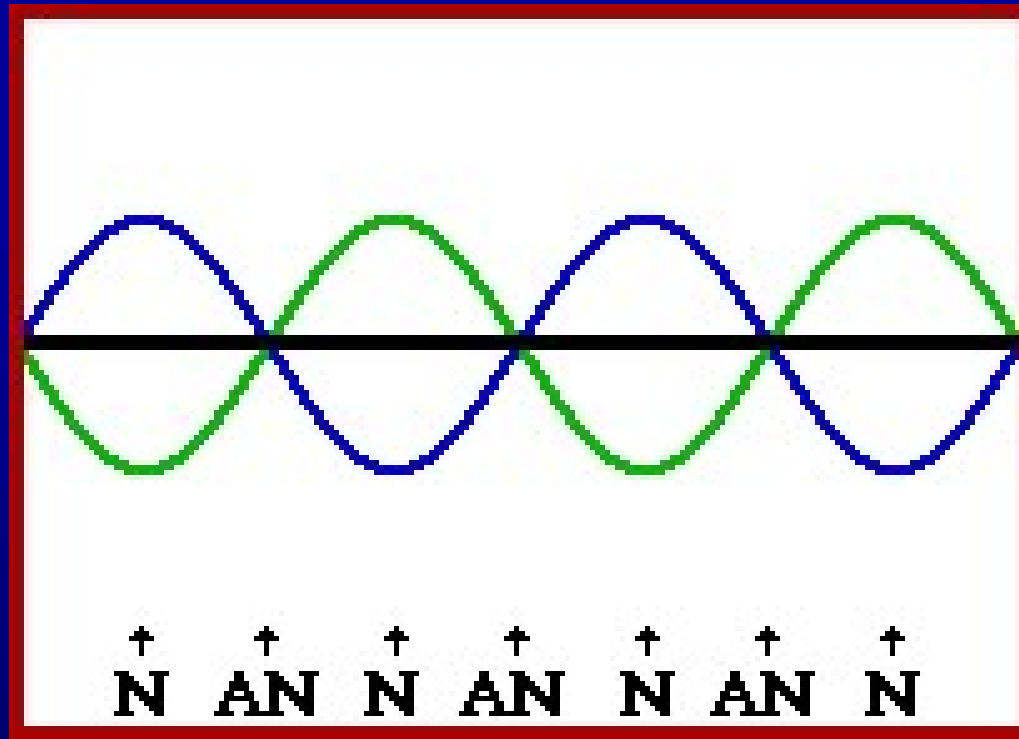


**reflection from (fixed)
hard boundary----**

half wavelength loss

$$u_1 = A \cos(kx - \omega t - \varphi)$$

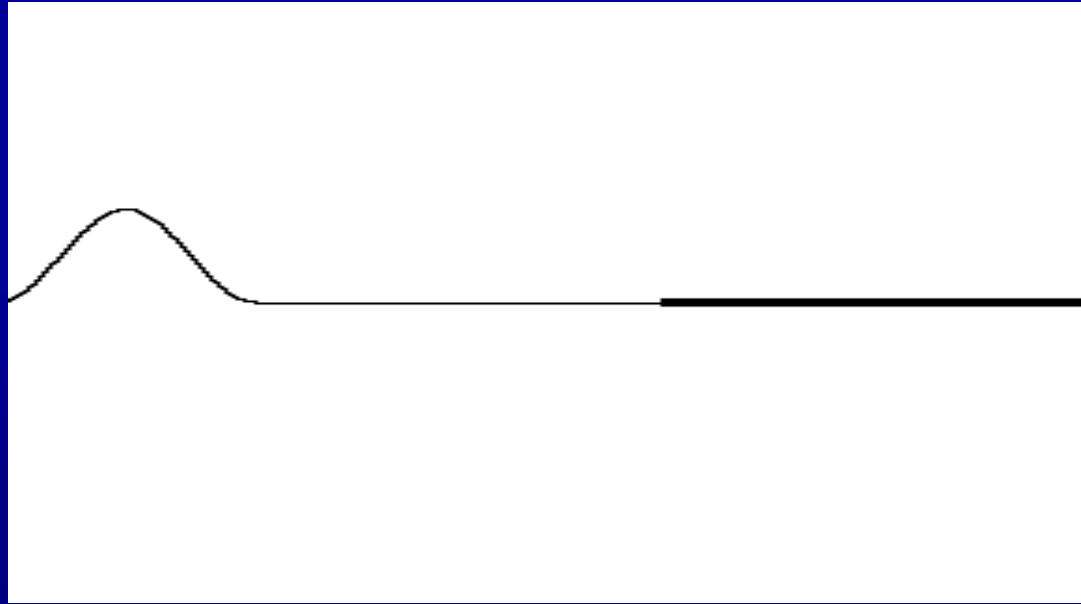
$$u_2 = A \cos(kx + \omega t)$$



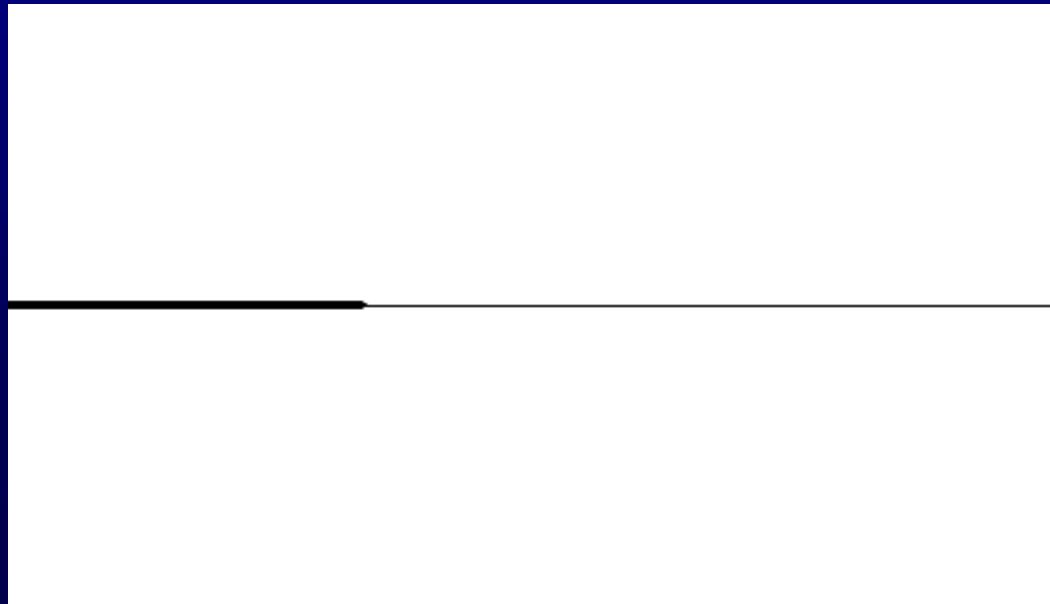
**reflection from free
(soft) boundary---**

**No half-wavelength
loss**

From high speed
to low speed (low
density to high
density)



From low speed to
high speed (high
density to low
density)



Longitudinal case

$$F_T \longrightarrow -pS$$

$$Y = \frac{L}{S} \left(\frac{\partial F_T}{\partial L} \right)$$

$$\longrightarrow -\frac{L}{S} \frac{\partial(pS)}{\partial L} = -V \left(\frac{\partial p}{\partial V} \right)_T \equiv B = \frac{1}{\kappa_T}$$

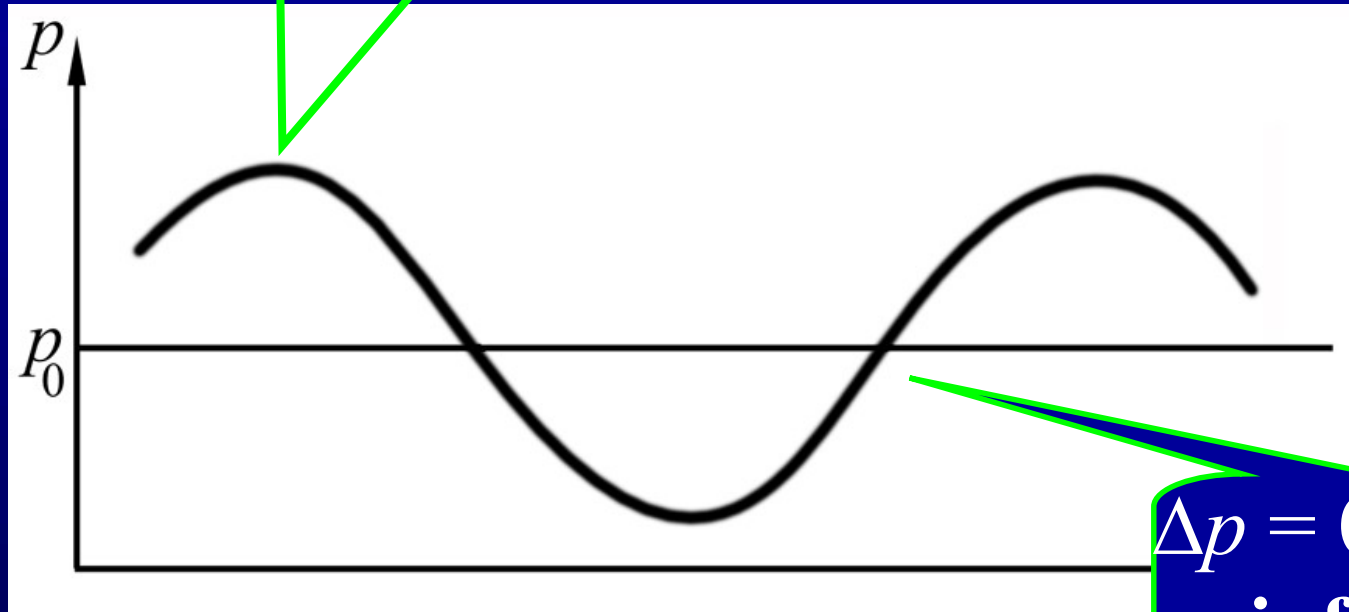
B **bulk modulus of elasticity**

κ_T **isothermal compressibility**

$$\Delta p = p - p_0$$

$$\Delta p = -B \frac{dV}{V} = -B \frac{\partial u}{\partial x}$$

$$\frac{\partial(\Delta p)}{\partial x} = -B \frac{\partial^2 u}{\partial x^2}$$



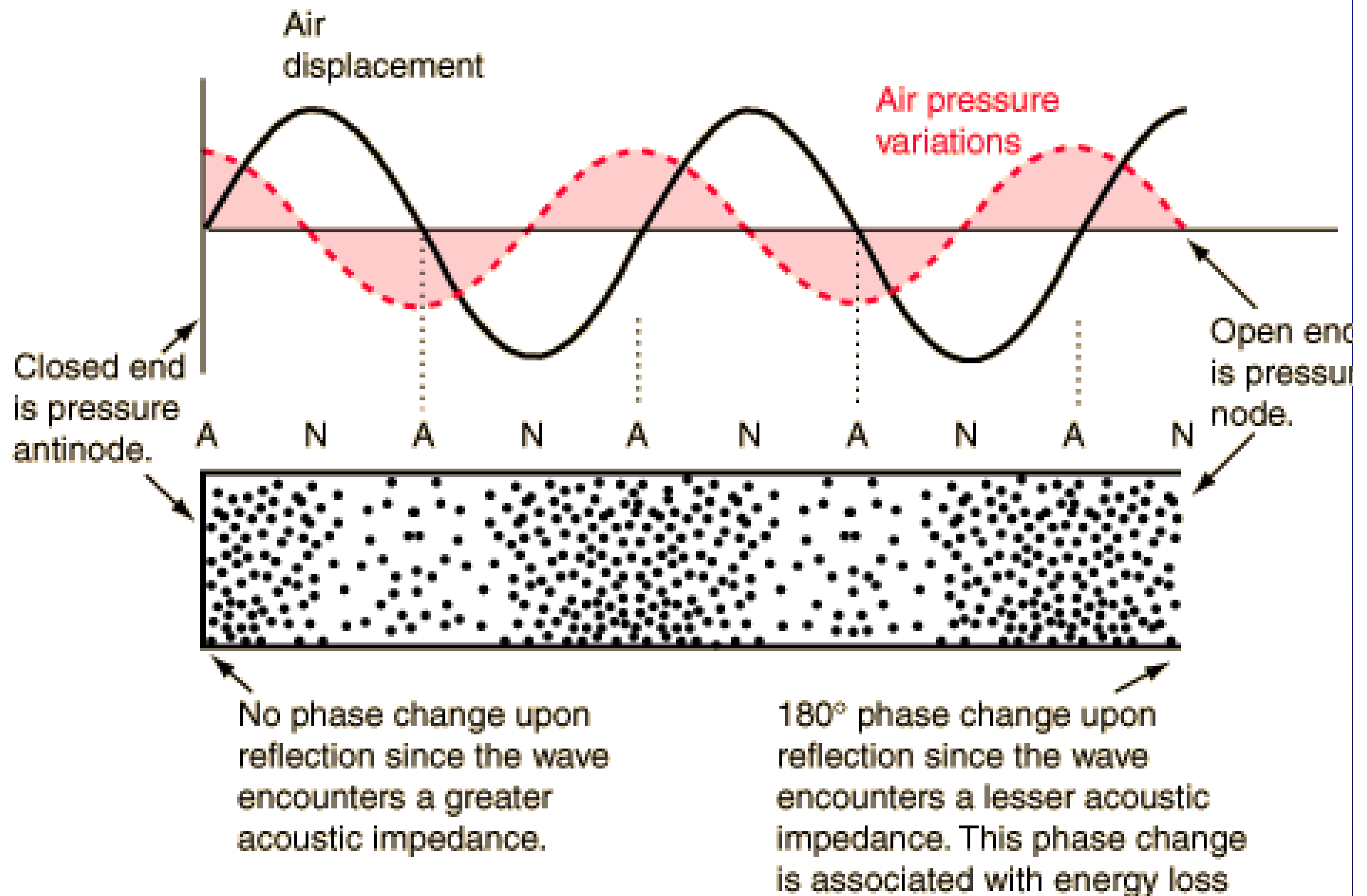
$\Delta p = 0$ node &
inflection

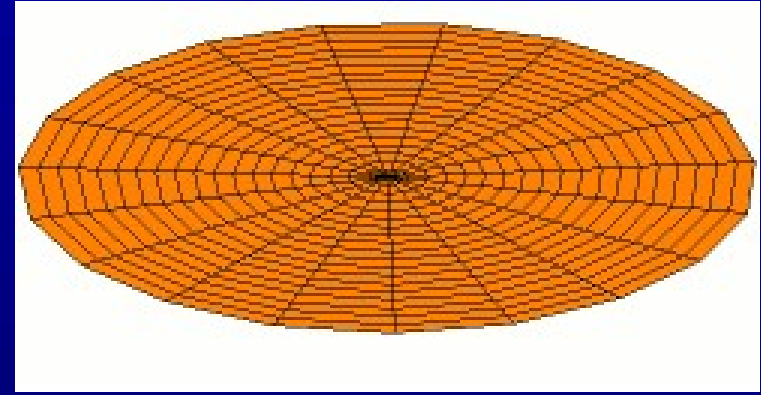
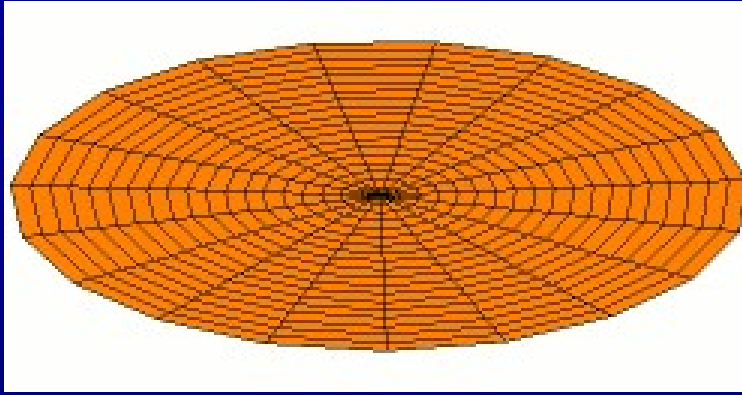
node 波节

inflection 波腹

$$\frac{\partial u}{\partial x} = 0$$

Production of a standing wave in an air column involves reflections from both the closed end and the open end of the column.





Examples of two-dimensional standing waves

right figure--two nodal lines(节线)crossing at the center

Energy transport

$$\begin{aligned} dE &= \frac{1}{2}(\rho \, dx) \left(\frac{\partial u}{\partial t} \right)^2 + F_T (ds - dx) \\ &= \frac{1}{2}(\rho \, dx) \left(\frac{\partial u}{\partial t} \right)^2 + F_T dx \left(\sqrt{1 + \left(\frac{\partial u}{\partial x} \right)^2} - 1 \right) \end{aligned}$$

Energy transport

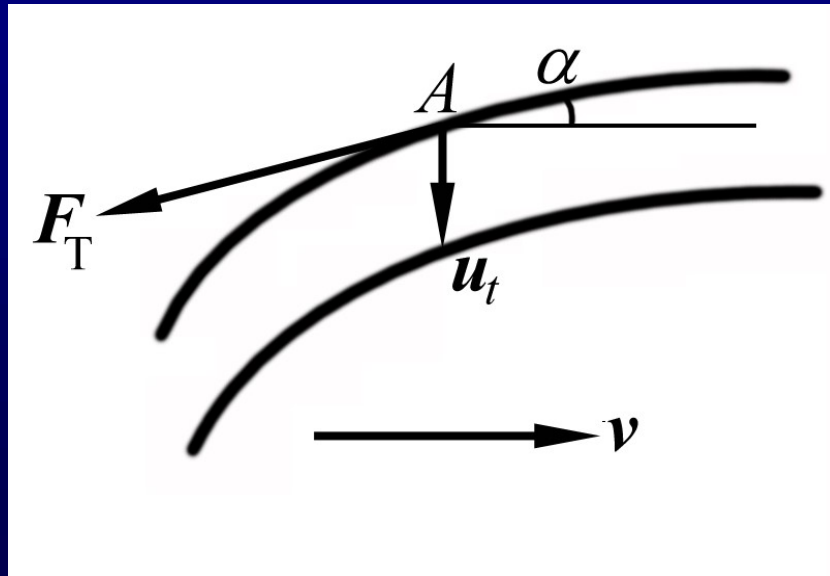
$$\begin{aligned} dE &= \frac{1}{2}(\rho \, dx) \left(\frac{\partial u}{\partial t} \right)^2 + F_T (ds - dx) \\ &\approx \frac{1}{2}(\rho \, dx) \left(\frac{\partial u}{\partial t} \right)^2 + \frac{1}{2} F_T dx \left(\frac{\partial u}{\partial x} \right)^2 \end{aligned}$$

Energy density

$$\begin{aligned} w &= \frac{1}{2} \rho \left[\left(\frac{\partial u}{\partial t} \right)^2 + v^2 \left(\frac{\partial u}{\partial x} \right)^2 \right] & F_T &= \rho v^2 \\ & & u_t &= \mp v u_x \end{aligned}$$

$$w = \rho \left(\frac{\partial u}{\partial t} \right)^2 = F_T \left(\frac{\partial u}{\partial x} \right)^2$$

Energy flow For a right-traveling wave
(wave source is at lhs), we have



$$u_t = -v u_x$$

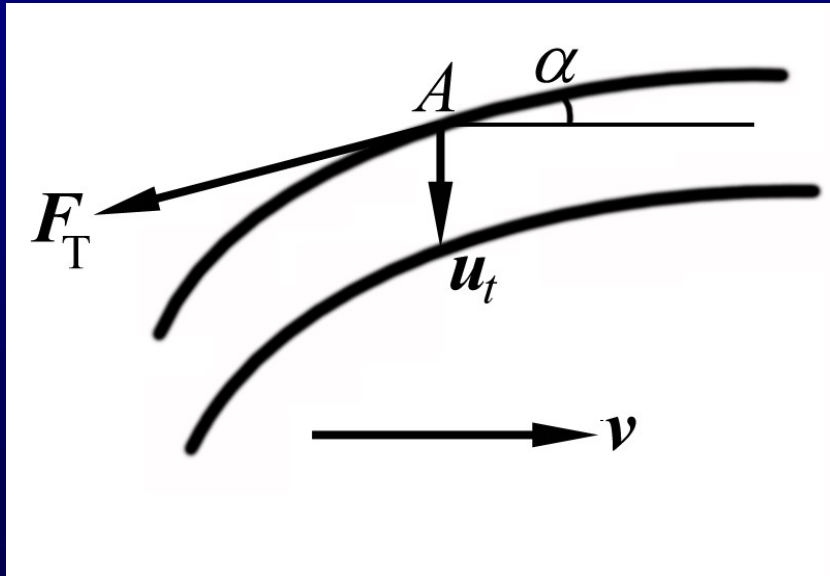
For ascending section
 $u_x > 0$, so $u_t < 0$ as
shown in the figure.

$$P = \mathbf{F} \cdot \frac{\partial \mathbf{u}}{\partial t} = -F_T \sin \alpha \frac{\partial u}{\partial t}$$

$$= -F_T \frac{\partial u}{\partial x} \frac{\partial u}{\partial t}$$

$$= \pm F_T \left(\frac{\partial u}{\partial x} \right)^2 v$$

$$= \pm w v$$



$$u = A \cos(kx - \omega t - \varphi)$$

$$w = \rho \left(\frac{\partial u}{\partial t} \right)^2 = F_T \left(\frac{\partial u}{\partial x} \right)^2$$

$$P = \pm F_T A^2 k^2 \sin^2(kx - \omega t - \varphi) v$$

$$= \pm \rho A^2 \omega^2 \sin^2(kx - \omega t - \varphi) v$$

$$\bar{P} = \frac{1}{T} \int_0^T P dt = \pm \frac{1}{2} \rho A^2 \omega^2 v \sim A^2$$

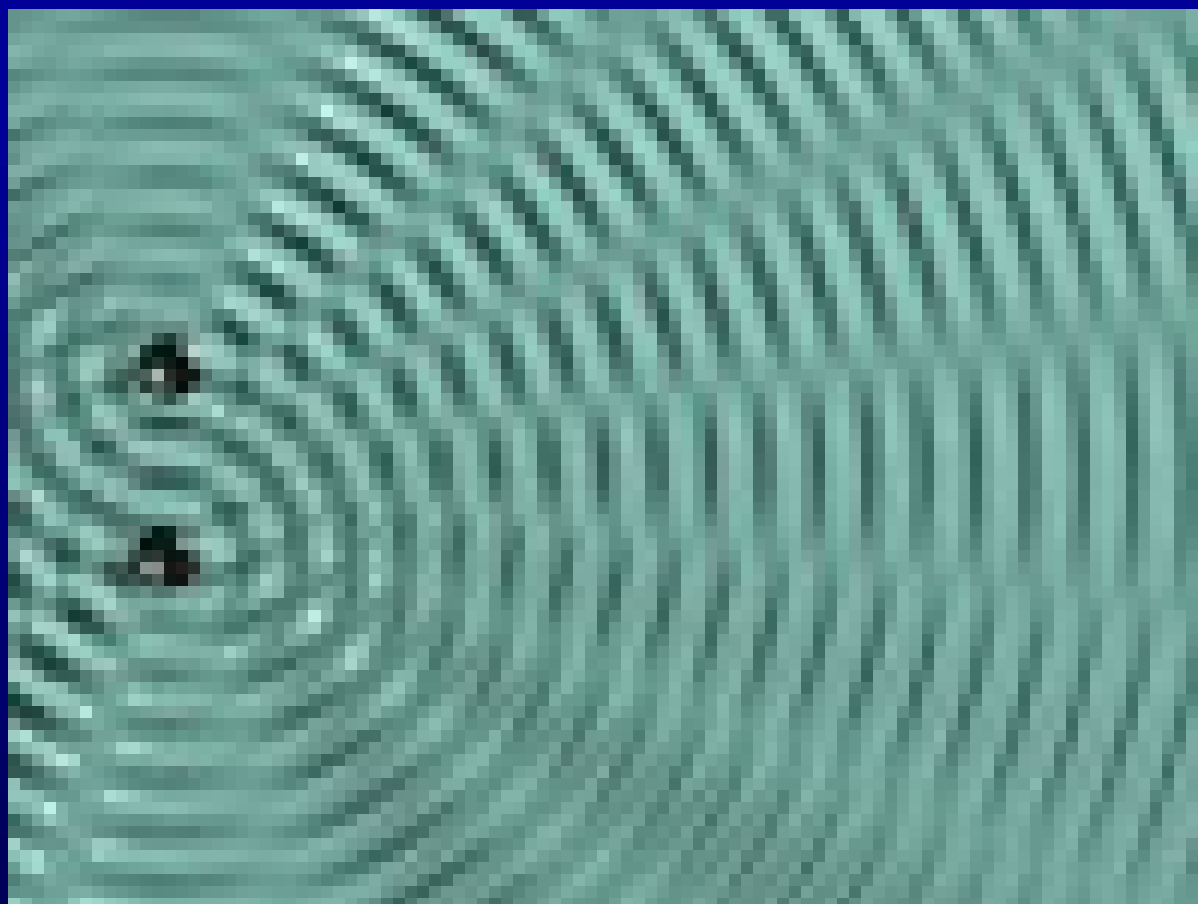
3D wave intensity

$$I \equiv \frac{|\bar{P}|}{S}$$

8.4 Interference and diffraction

干涉与衍射





$$u_1 = A \cos(kx - \omega t - \varphi)$$

$$u_2 = A \cos(kx - \omega t)$$

φ may be caused by path difference Δx :

$$kx - \varphi = k \left(x - \frac{\varphi}{k} \right) \equiv k(x - \Delta x)$$

$$u_1 + u_2 = \underbrace{2A \cos \frac{\varphi}{2}} \cos \left(kx - \omega t - \frac{\varphi}{2} \right)$$

Constructive or destructive interference

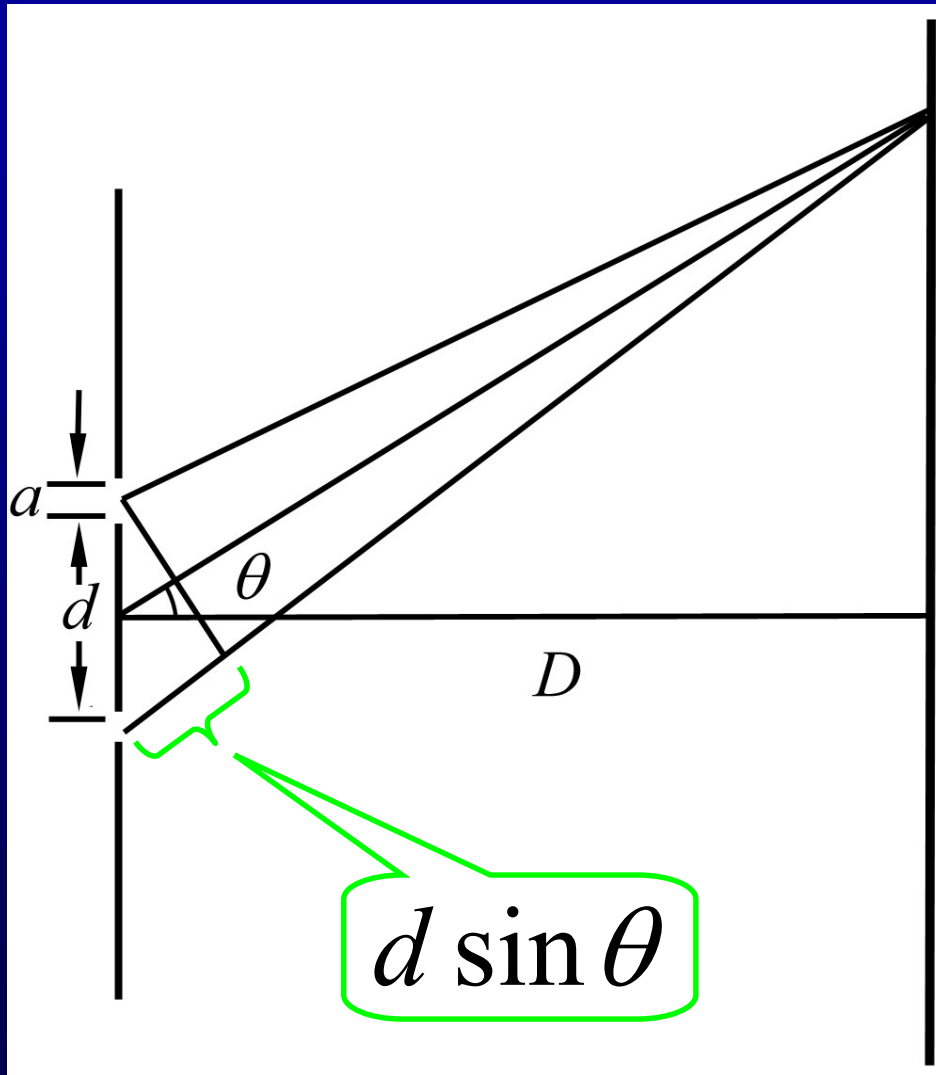
$$I \propto \cos^2 \frac{\varphi}{2} = \begin{cases} 1, & \varphi = 2m\pi, & \Delta x = m\lambda \\ 0, & \varphi = (2m+1)\pi, & \Delta x = (m+1/2)\lambda \end{cases}$$

Corresponding path difference to constructive interference is

$$\Delta x \equiv \frac{\varphi}{k} = \frac{2m\pi}{\frac{2\pi}{\lambda}} = m\lambda$$

Young's experiment

$$a \ll \lambda \quad d \ll D$$



Bright and dark fringes appear at

$$d \sin \theta = m \lambda$$

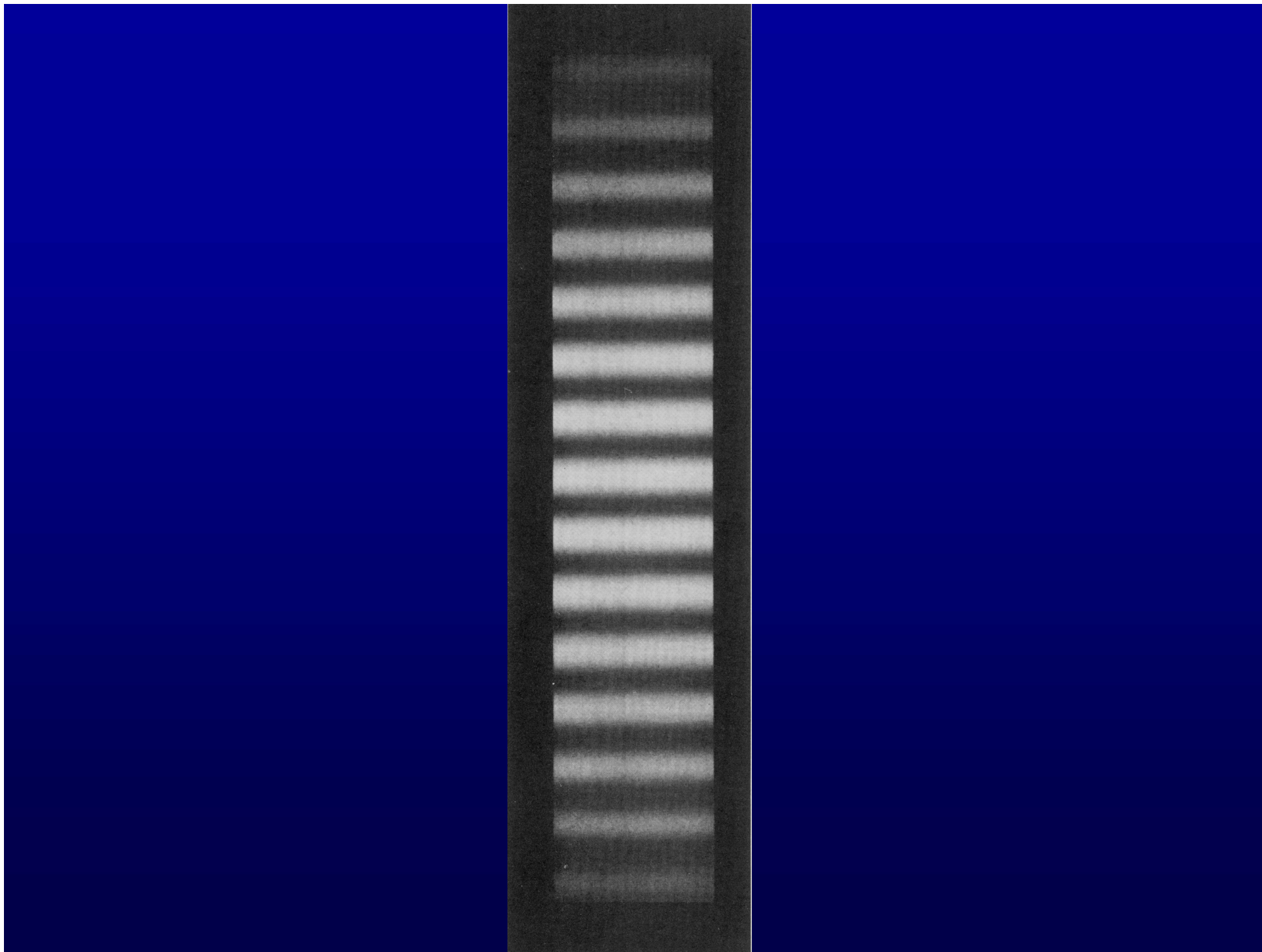
$$m = 0, 1, 2, \dots$$

$$d \sin \theta = \left(m + \frac{1}{2} \right) \lambda$$

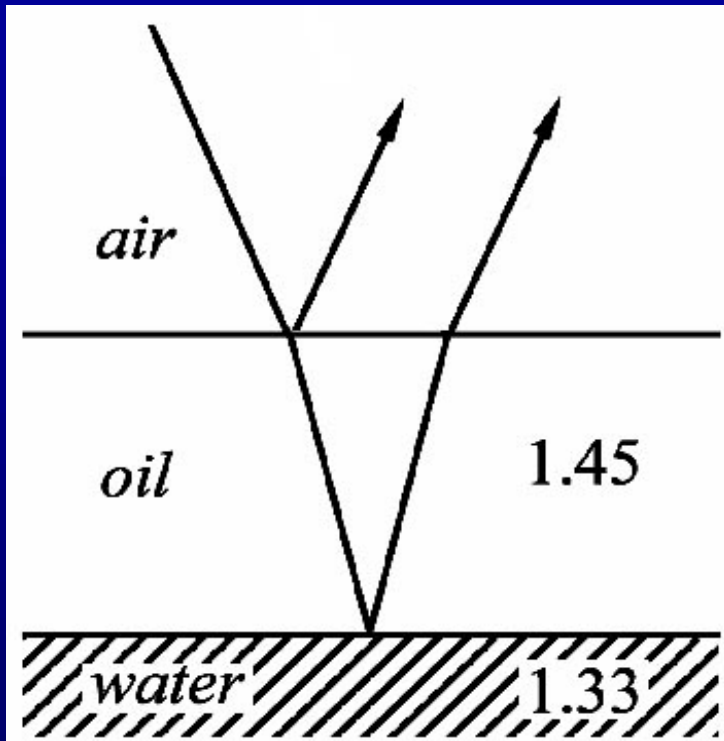
$$m = 0, 1, 2, \dots$$

Rosetta stone

Egyptian hieroglyphs



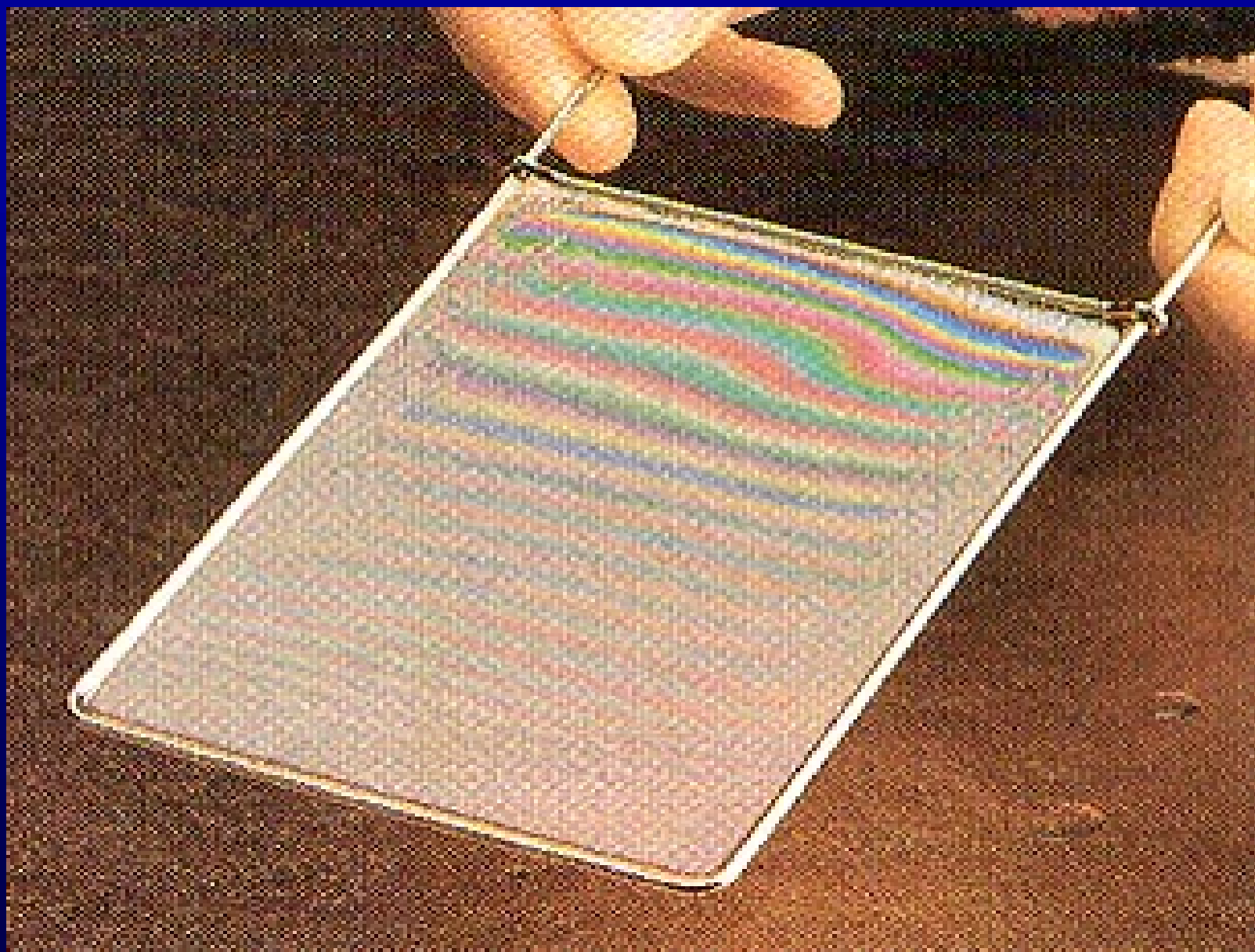
Film interference

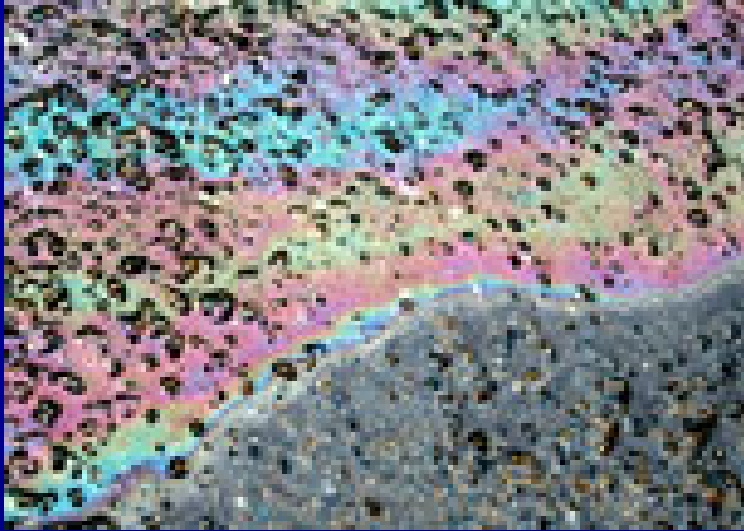


phase difference π
(optical path difference $\lambda/2$)
at the air-oil interface

Maxima at

$$2nd = \left(m + \frac{1}{2} \right) \lambda$$





gasoline on water



opals (猫眼石)

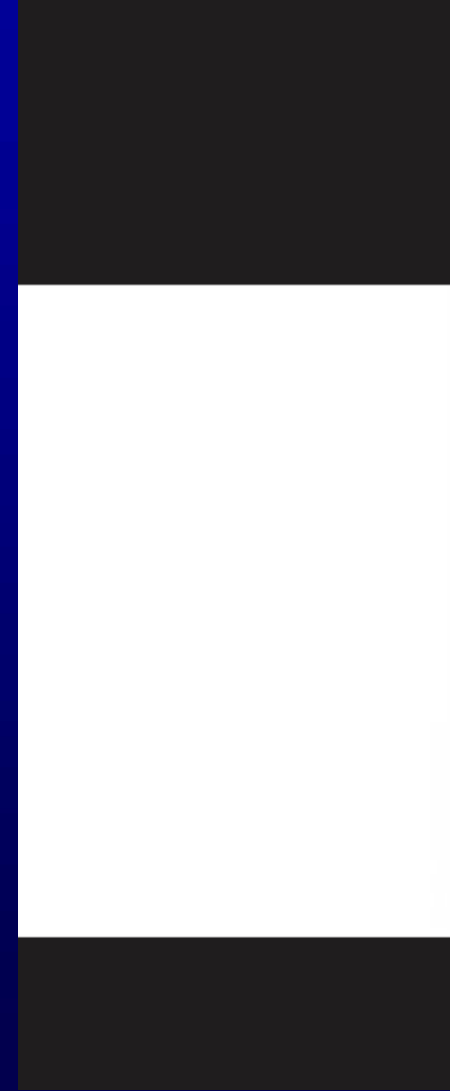
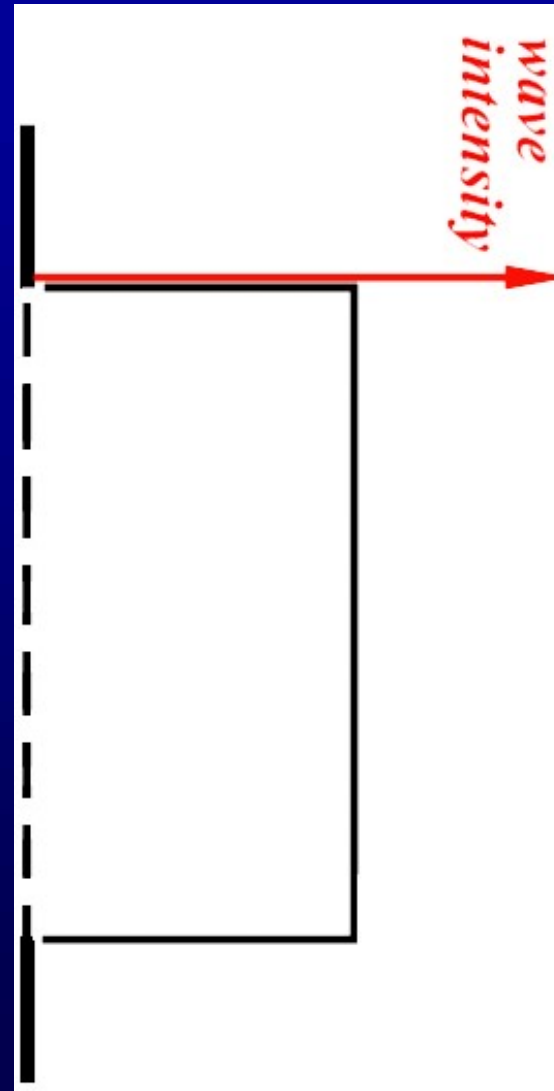
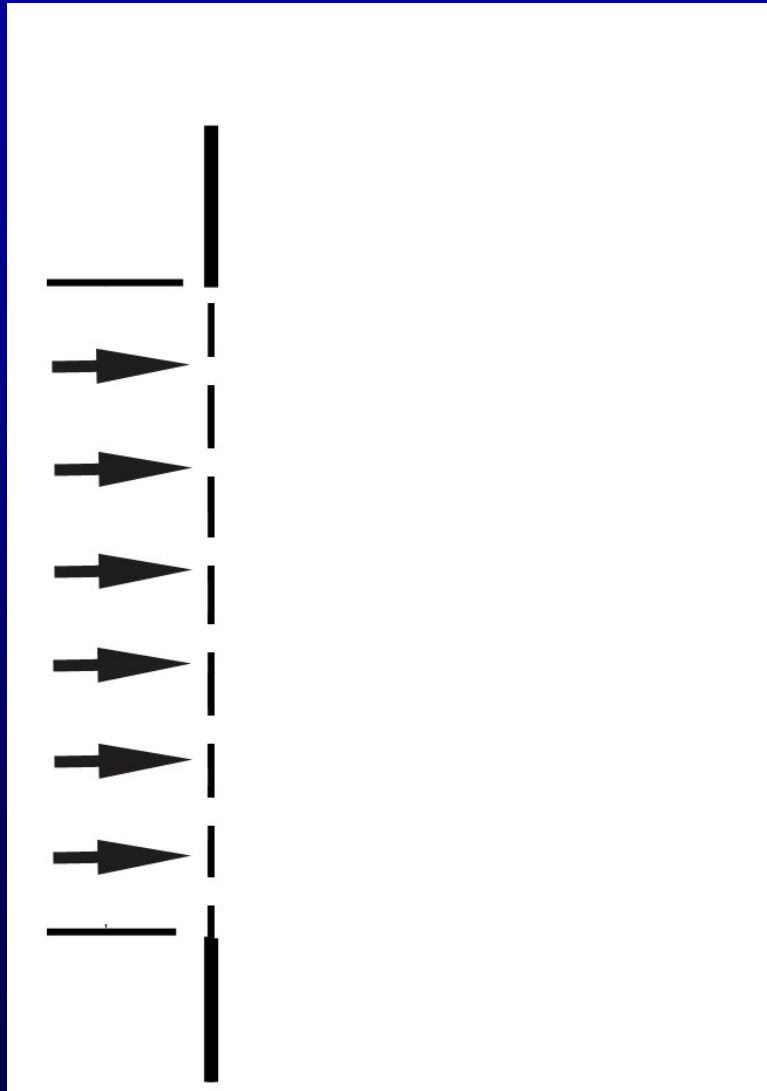


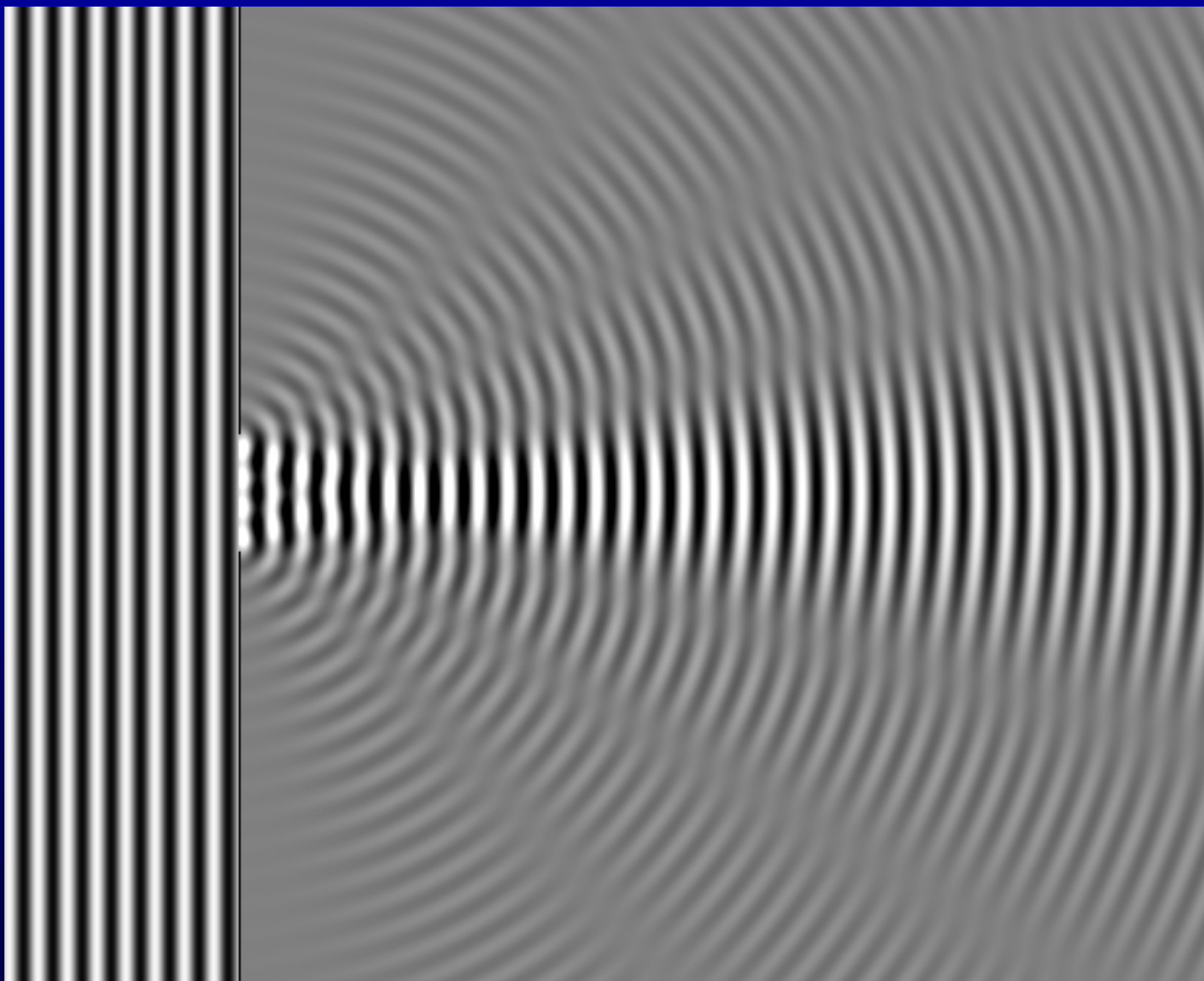
butterfly wings



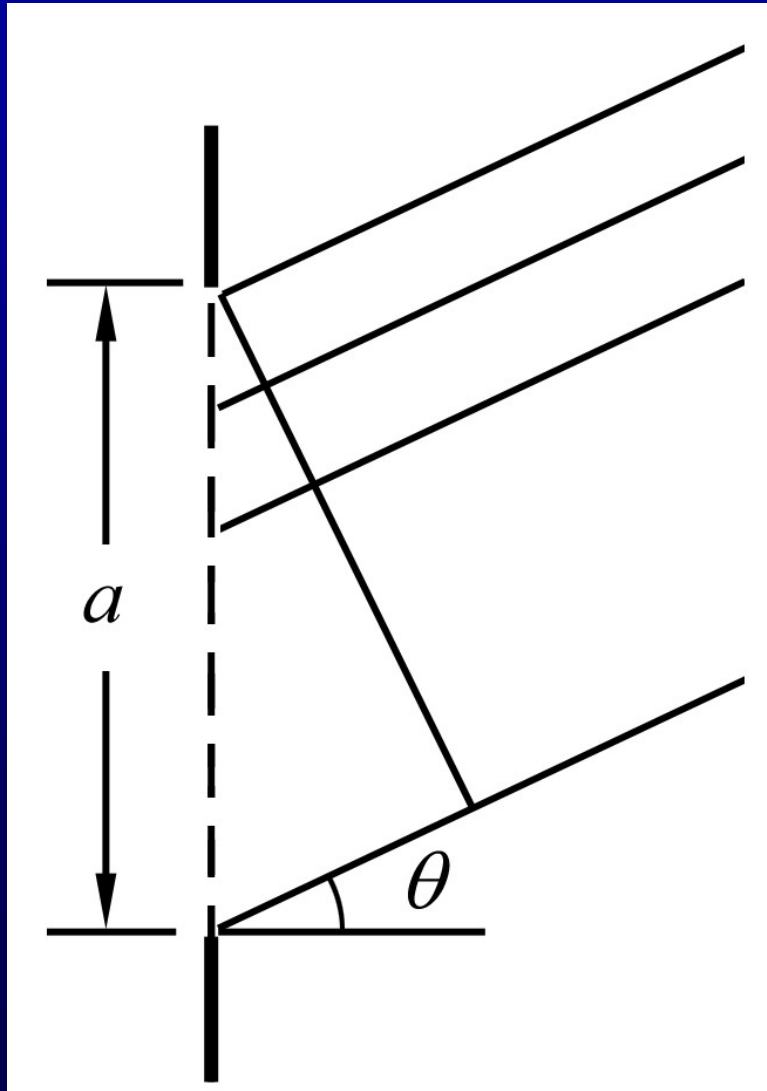
pigeon feathers

Diffraction





diffraction



Phase difference

$$\Delta\varphi = k\Delta x = \underline{k} \frac{n}{N} \underline{a \sin \theta} \equiv \frac{n}{N} \underline{\varphi}$$

$$U_{\theta} = \sum_{n=0}^{N-1} u_n(\theta)$$

$$= \sum_{n=0}^{N-1} A \cos\left(kx - \omega t + \frac{n}{N} \varphi\right)$$

$$U_0 = NA \cos(kx - \omega t) \\ \equiv A_0 \cos(kx - \omega t)$$

measured amplitude

$$U_{\theta} = \sum_{n=0}^{N-1} u_n(\theta)$$

$$= \sum_{n=0}^{N-1} \Delta n \cdot \frac{A_0}{N} \cos\left(kx - \omega t + \frac{n}{N} \varphi\right)$$

$$= \sum_{n=0}^{N-1} \Delta \left(\frac{n}{N}\right) A_0 \cos\left(kx - \omega t + \frac{n}{N} \varphi\right)$$

$$\rightarrow \int_0^1 dy A_0 \cos(kx - \omega t + y \varphi)$$

$$= \frac{A_0}{\varphi} [\sin(kx - \omega t + \varphi) - \sin(kx - \omega t)]$$

$$= \frac{A_0}{\varphi} [\sin(kx - \omega t + \varphi) - \sin(kx - \omega t)]$$

$$= \frac{2A_0}{\varphi} \sin\left(\frac{\varphi}{2}\right) \cos\left(kx - \omega t + \frac{\varphi}{2}\right)$$

$$= A_0 \frac{\sin\left(\frac{\varphi}{2}\right)}{\frac{\varphi}{2}} \cos\left(kx - \omega t + \frac{\varphi}{2}\right)$$

$$I = I_0 \frac{\sin^2(\varphi/2)}{(\varphi/2)^2} \quad \varphi = ka \sin \theta$$

$$I = I_0 \frac{\sin^2(\varphi/2)}{(\varphi/2)^2}$$

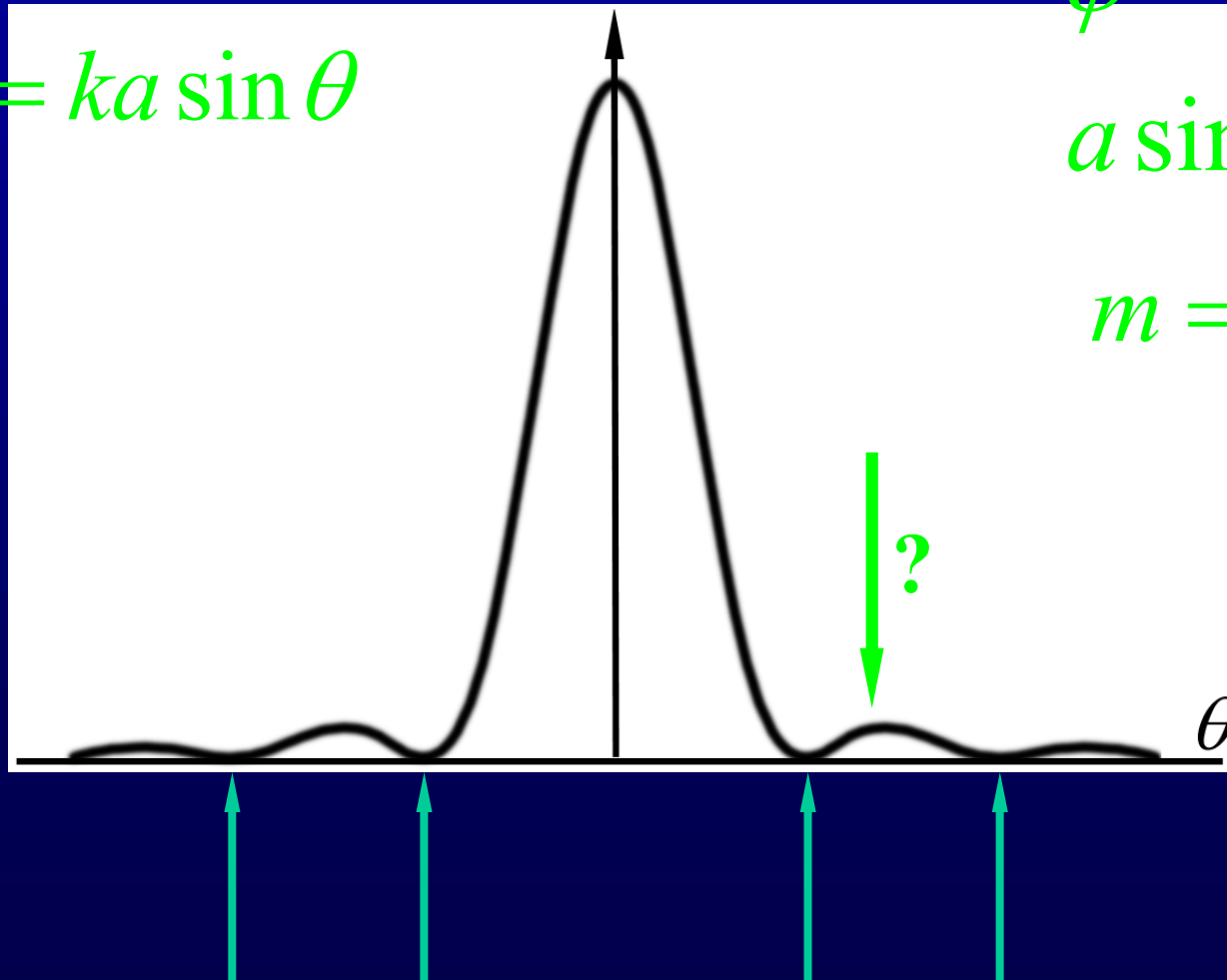
The **minima** occur at

$$\varphi = 2m\pi$$

$$a \sin \theta = m\lambda$$

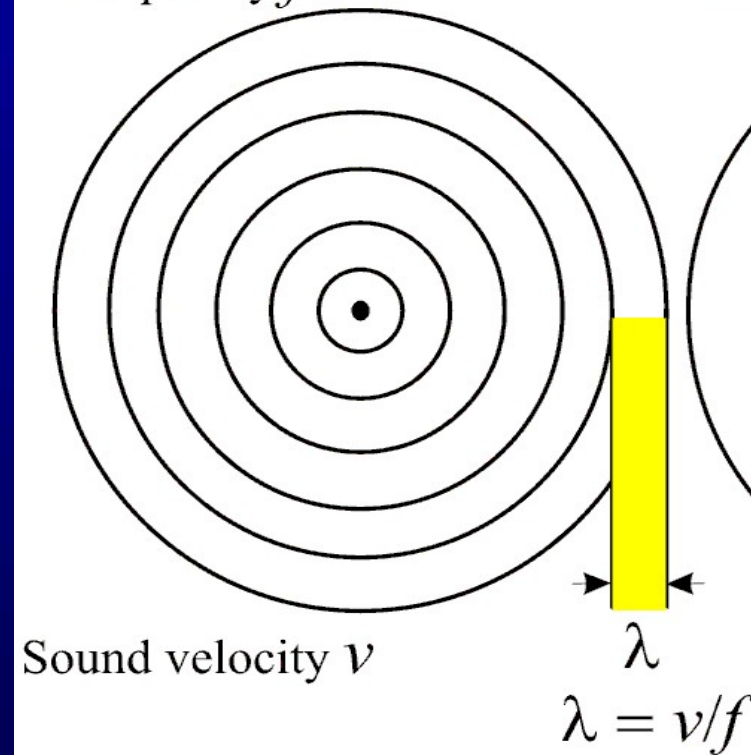
$$m = 1, 2, 3 \dots$$

$$\varphi = ka \sin \theta$$



8.6 Doppler effect

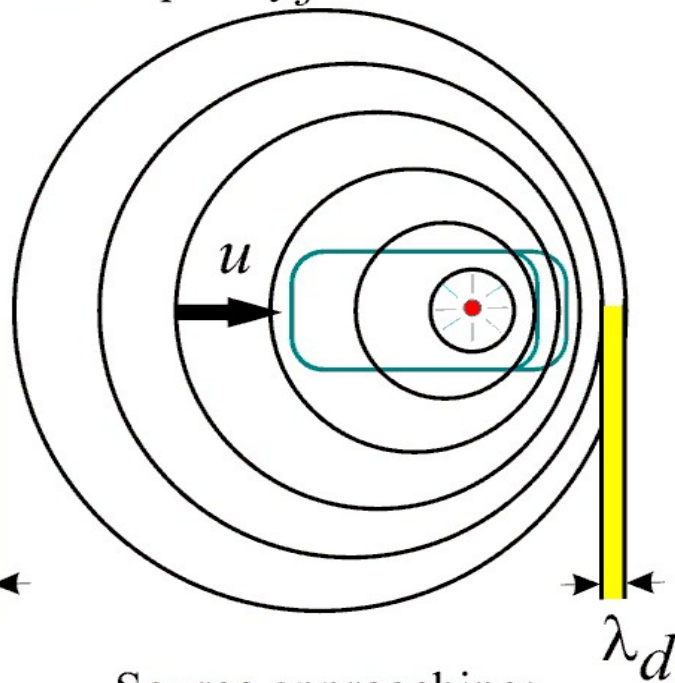
Stationary source
of frequency f



Sound velocity v

$$\lambda = v/f$$

Moving source
of frequency f



Source velocity u

Source approaching:

In period $1/f$, source moves

closer by u/f , so: $\lambda_d = (v - u)/f$

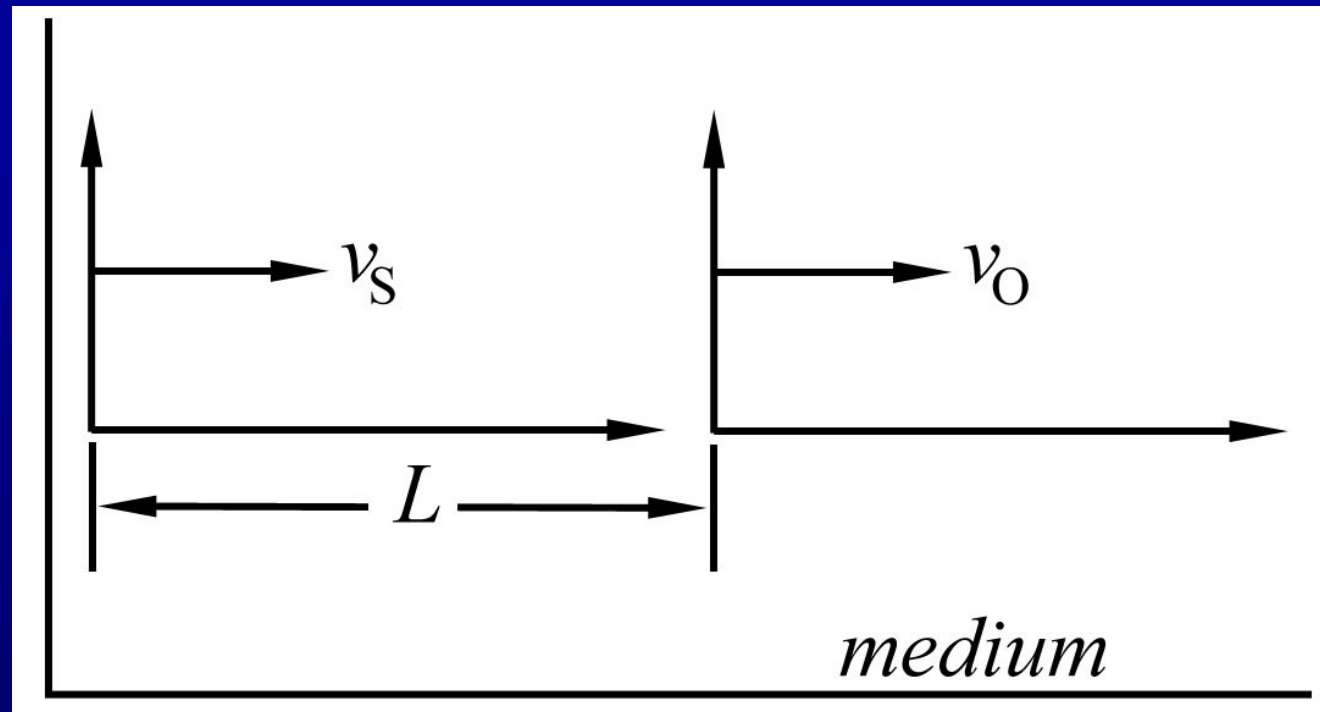
In a period $\frac{1}{f}$, the source moves closer $\frac{v_s}{f}$.

So, the wavelength observed become

$$\lambda' = \frac{v_p}{f} - \frac{v_s}{f}.$$

Therefore, the perceived frequency is

$$f' = \frac{v_p}{\lambda'} = f \frac{v_p}{v_p - v_s}$$



Signals issued at

$$x = 0, \quad t = 0$$

$$x = v_S T, \quad t = T$$

received at

$$L + v_O t'_1, \quad t'_1$$

$$L + v_O t'_2, \quad t'_2$$



$$v_p t'_1 = L + v_o t'_1$$

$$t'_1 = \frac{L}{v_p - v_o}$$

$$v_p (t'_2 - T) = L - (v_s - v_o)T + v_o (t'_2 - T)$$

$$t'_2 = \frac{L - (v_s - v_o)T}{v_p - v_o} + T$$

Thus, in the observer frame, the period is

$$T' = t'_2 - t'_1 = \frac{v_p - v_s}{v_p - v_o} T$$

if T happens to be the period of a wave in source frame.

$$\nu' = \frac{v_p - v_o}{v_p - v_s} \nu$$



$$\nu' = \frac{\nu_p - \nu_O}{\nu_p - \nu_S} \nu$$

$\nu_O > \nu_S$ (including $\nu_S \leq 0$) separating,

$\nu' < \nu$ Doppler redshift

$\nu_O < \nu_S$ (including $\nu_O \leq 0$) approaching,

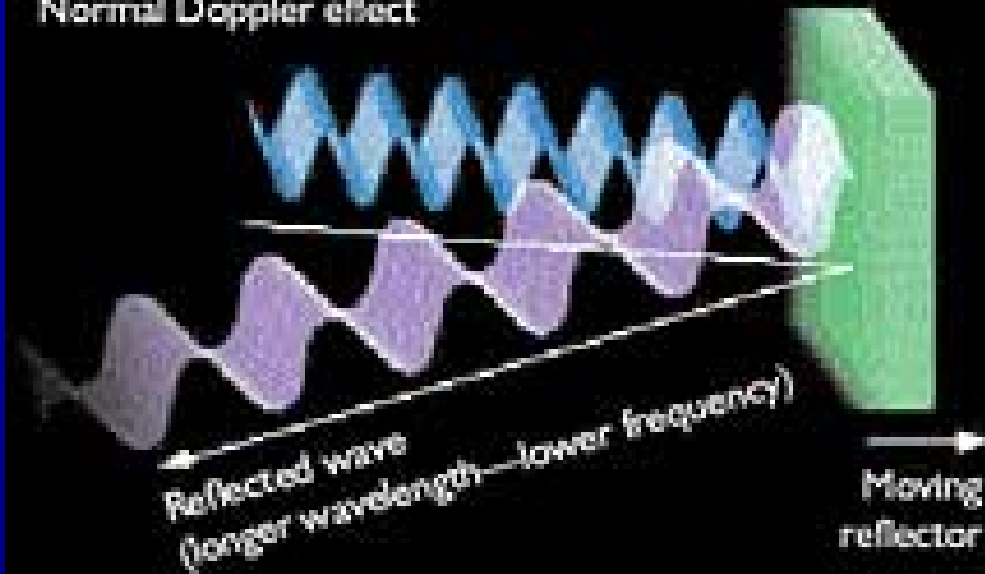
$\nu' > \nu$ Doppler blueshift

Inverse Doppler Demonstration Ends a 60-Year Quest

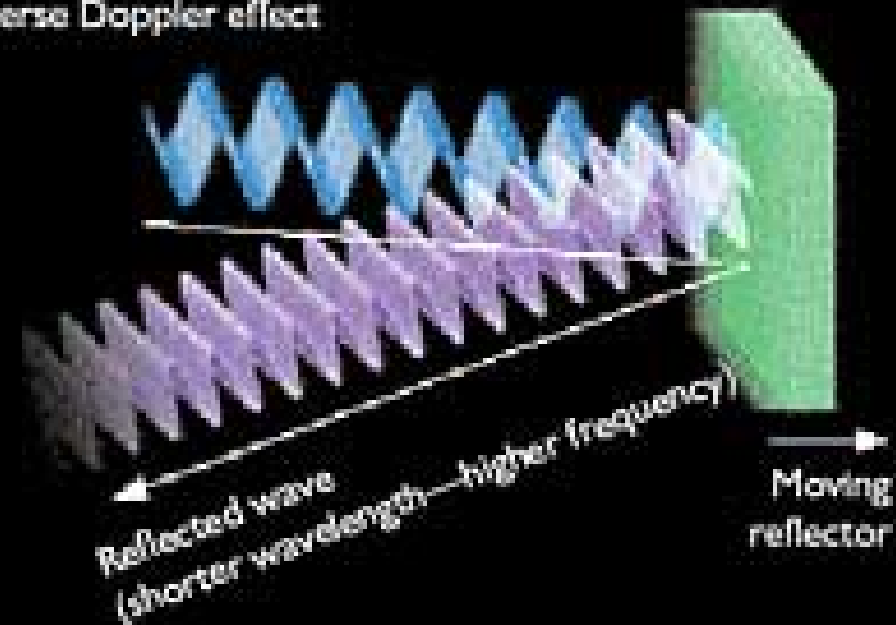
SCIENCE VOL 302 28 NOVEMBER 2003

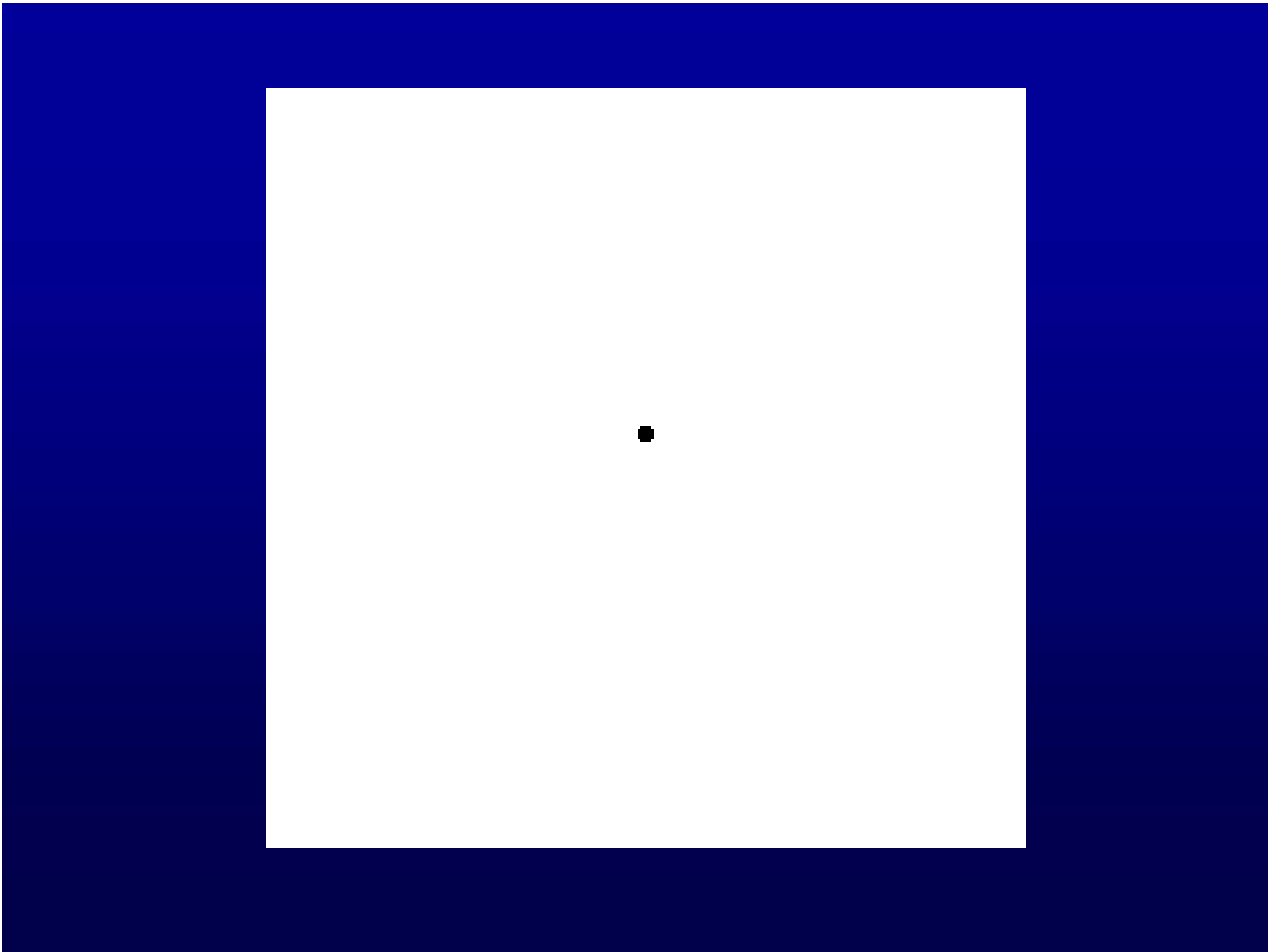
“To make that happen in reverse, the U.K. researchers needed to clear two big hurdles. First, they had to put a material into a state called anomalous dispersion that could support the specialized waves needed to produce an inverse Doppler effect. Then they had to generate a moving barrier within that material to reflect waves.”

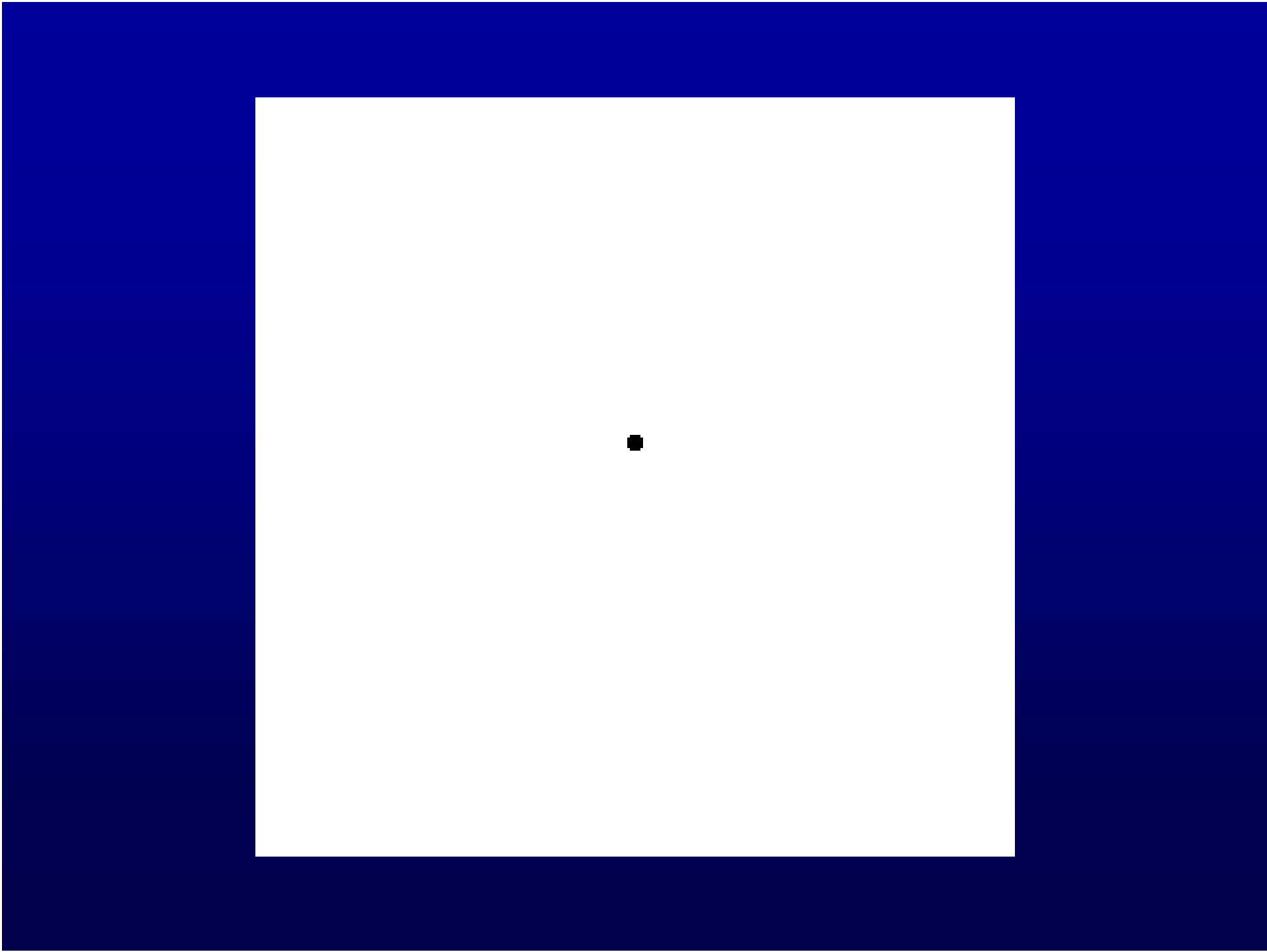
Normal Doppler effect

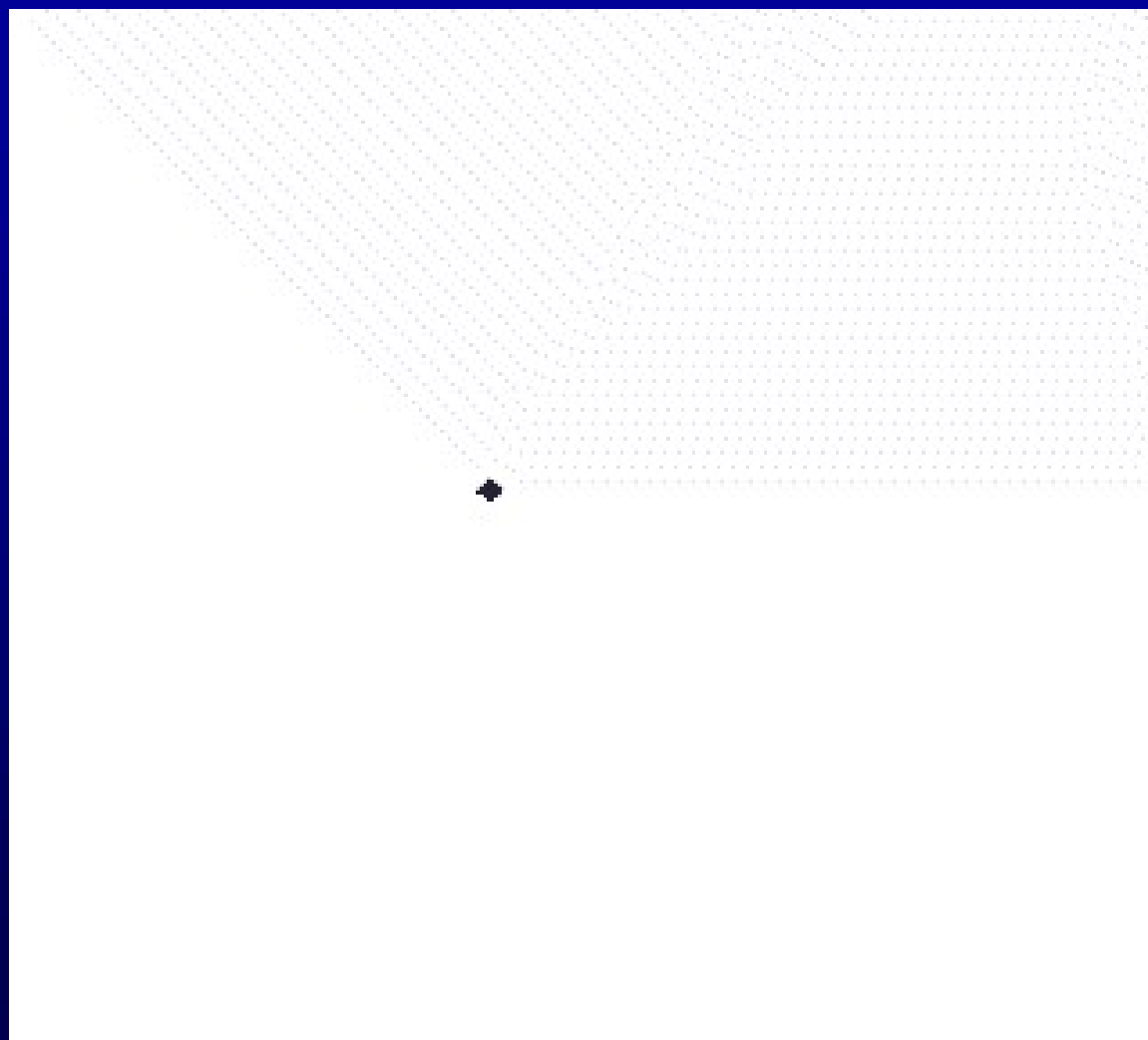


Inverse Doppler effect

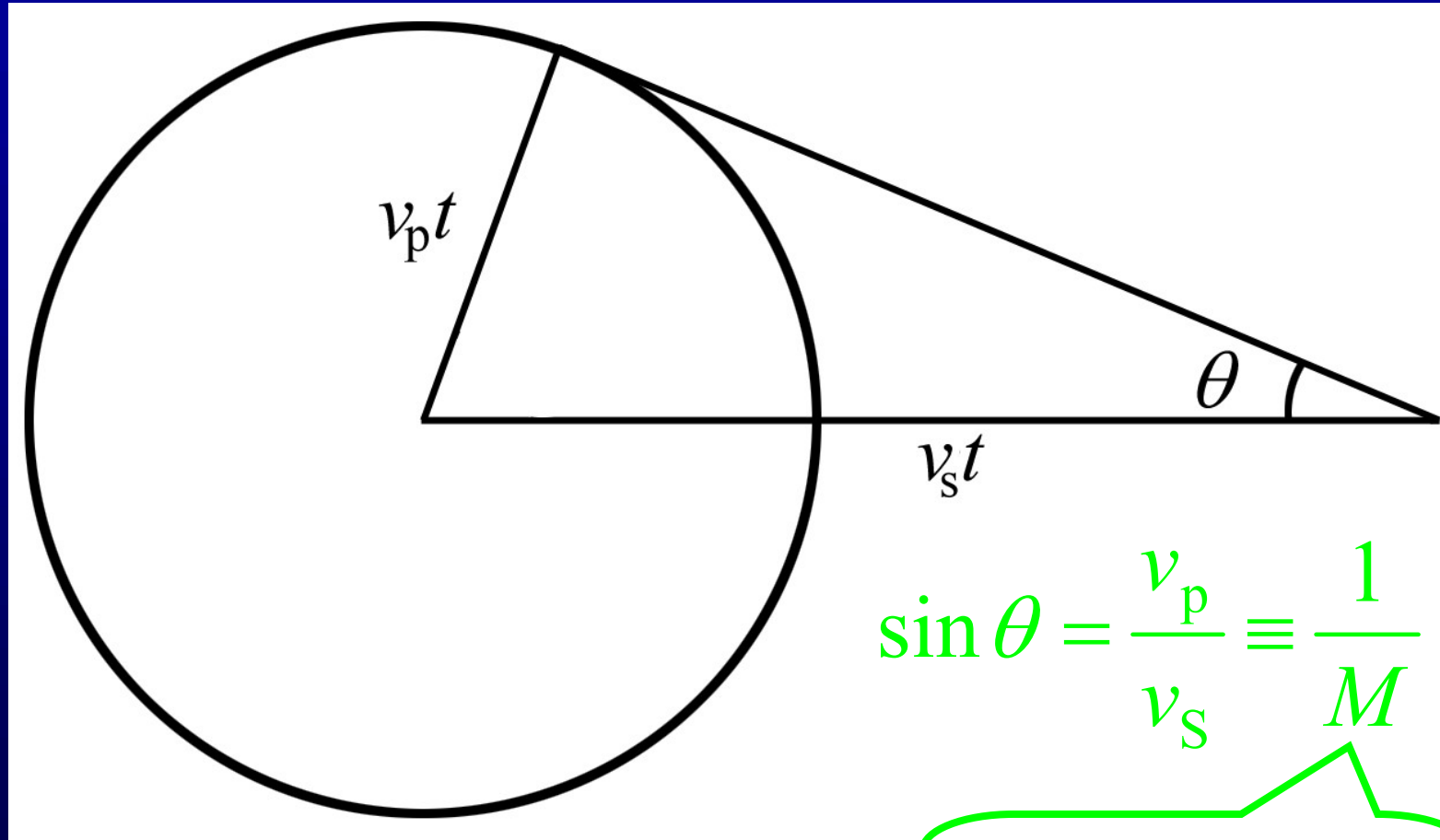






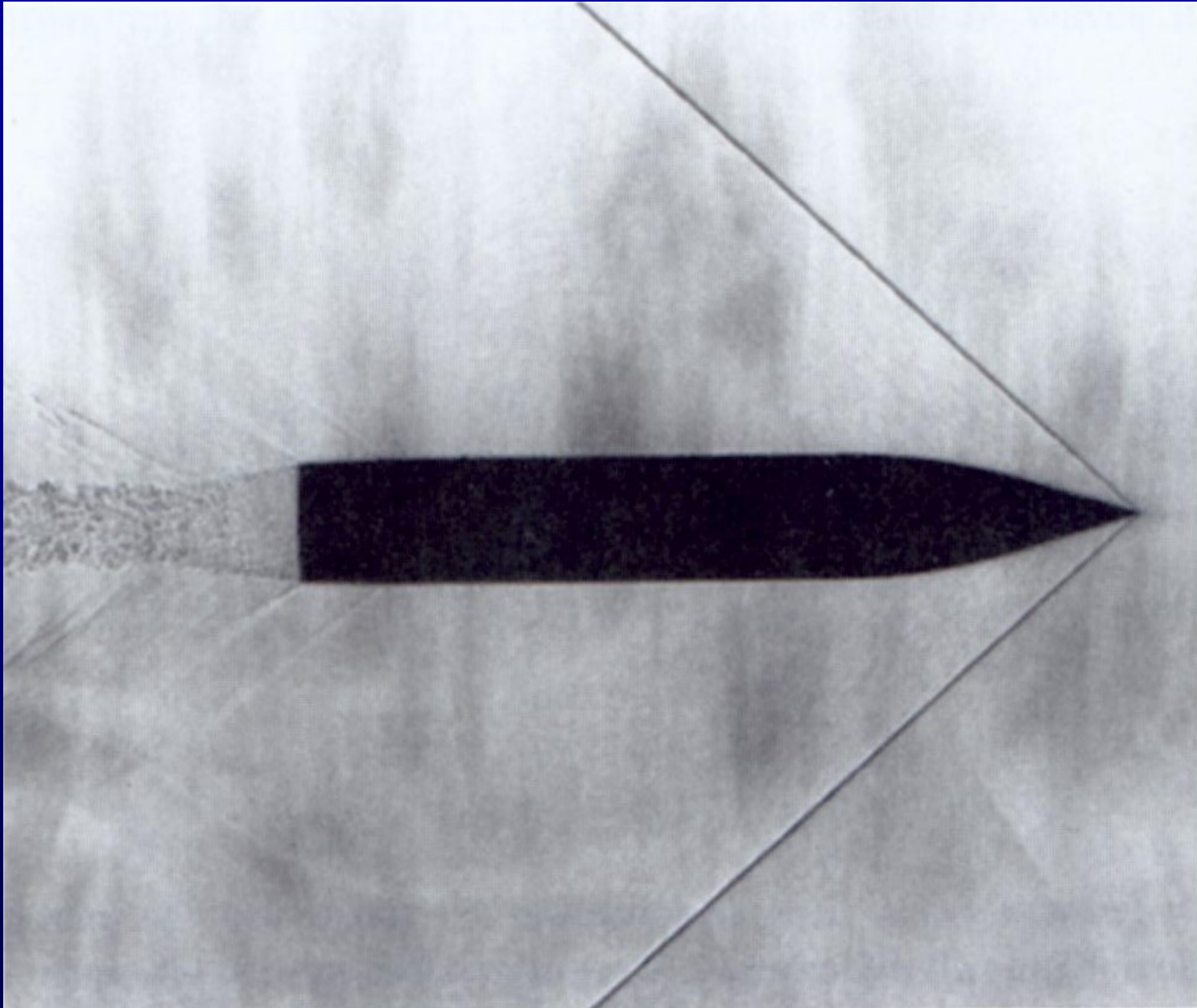


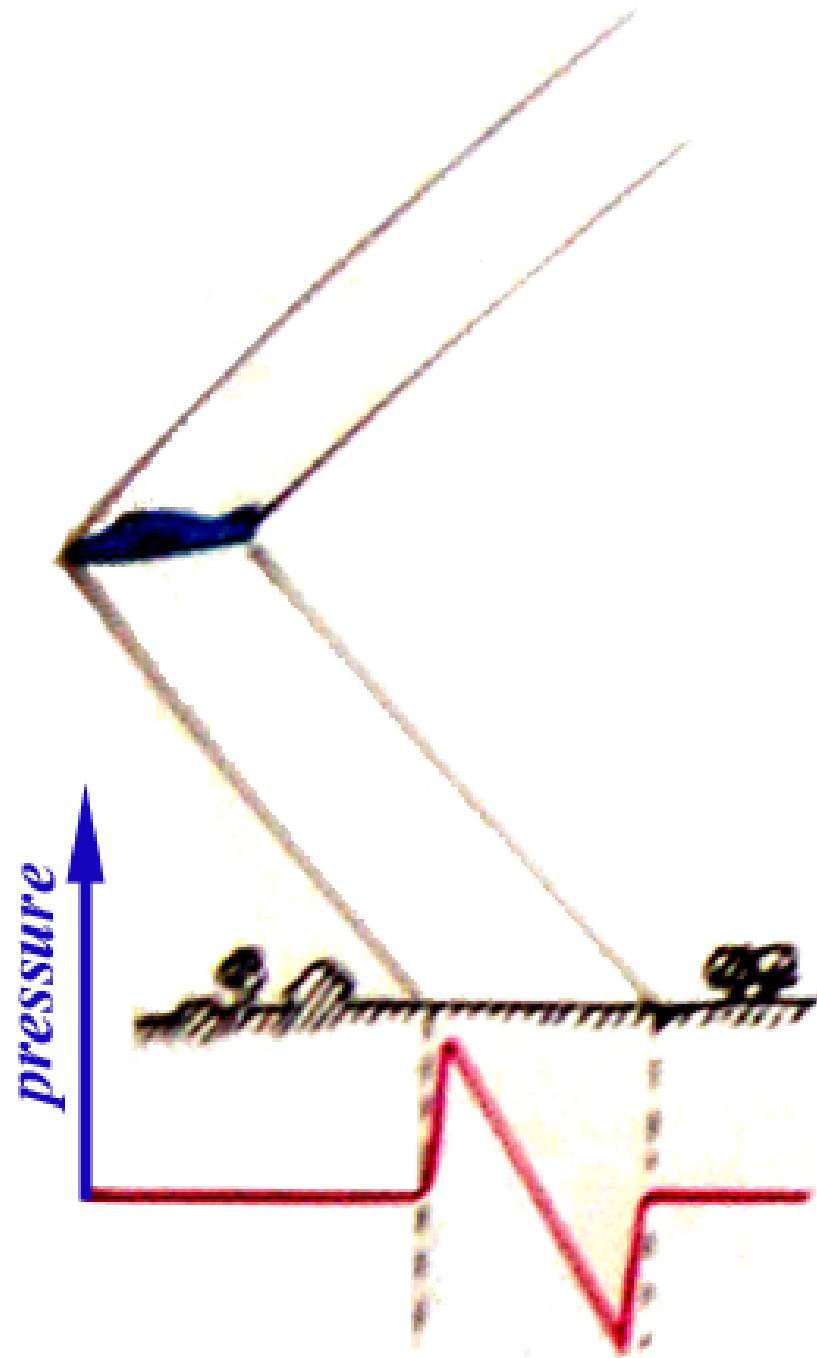
$v_s > v_p$ supersonic speed Mach cone
超音速



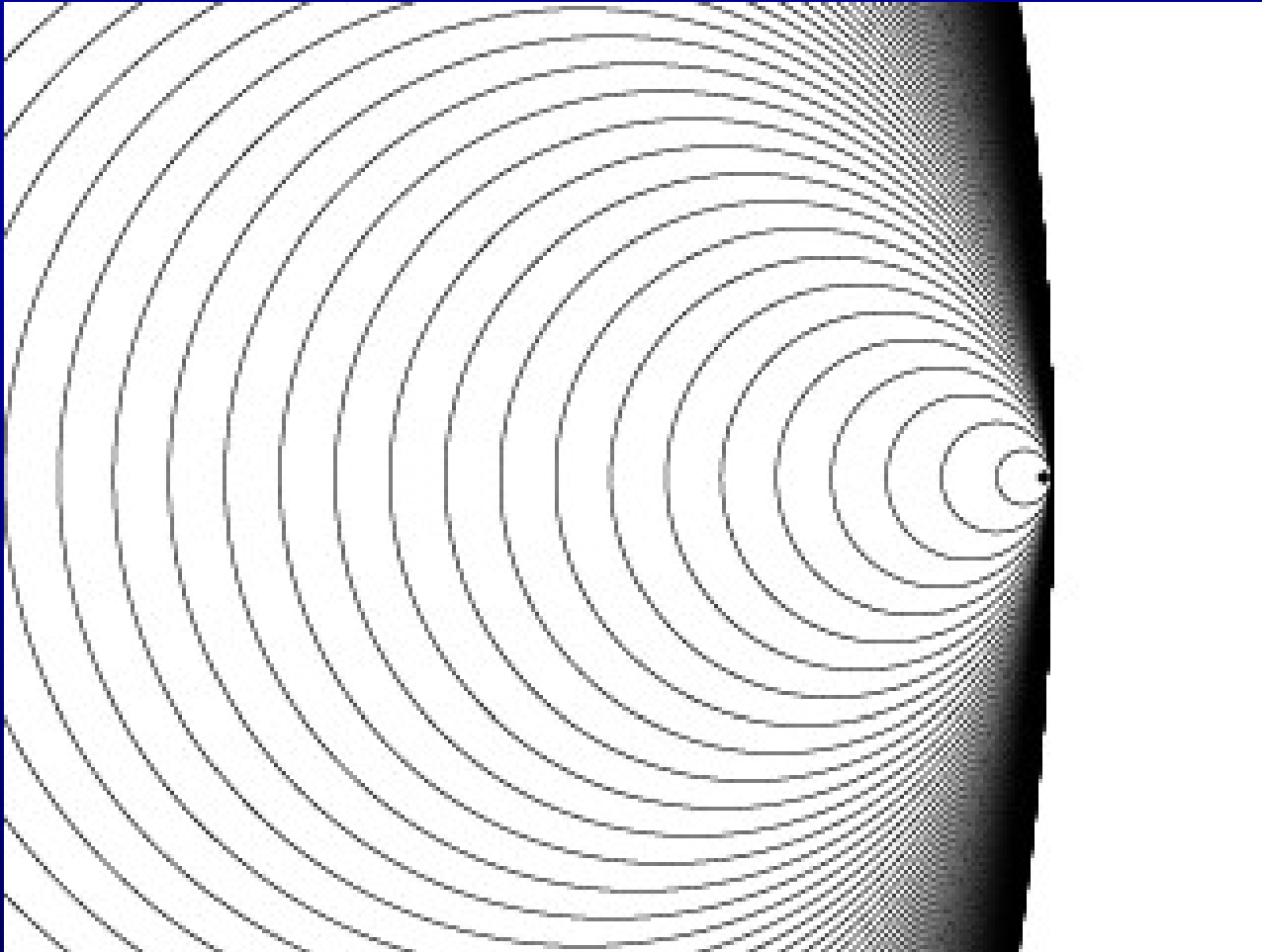
$$\sin \theta = \frac{v_p}{v_s} \equiv \frac{1}{M}$$

Mach number
in aerodynamics





‘Sonic boom’
shock wave



When the speed of a source equals the speed of sound , the resulting pile of waves forms a large amplitude "sound barrier" or "sonic barrier" .

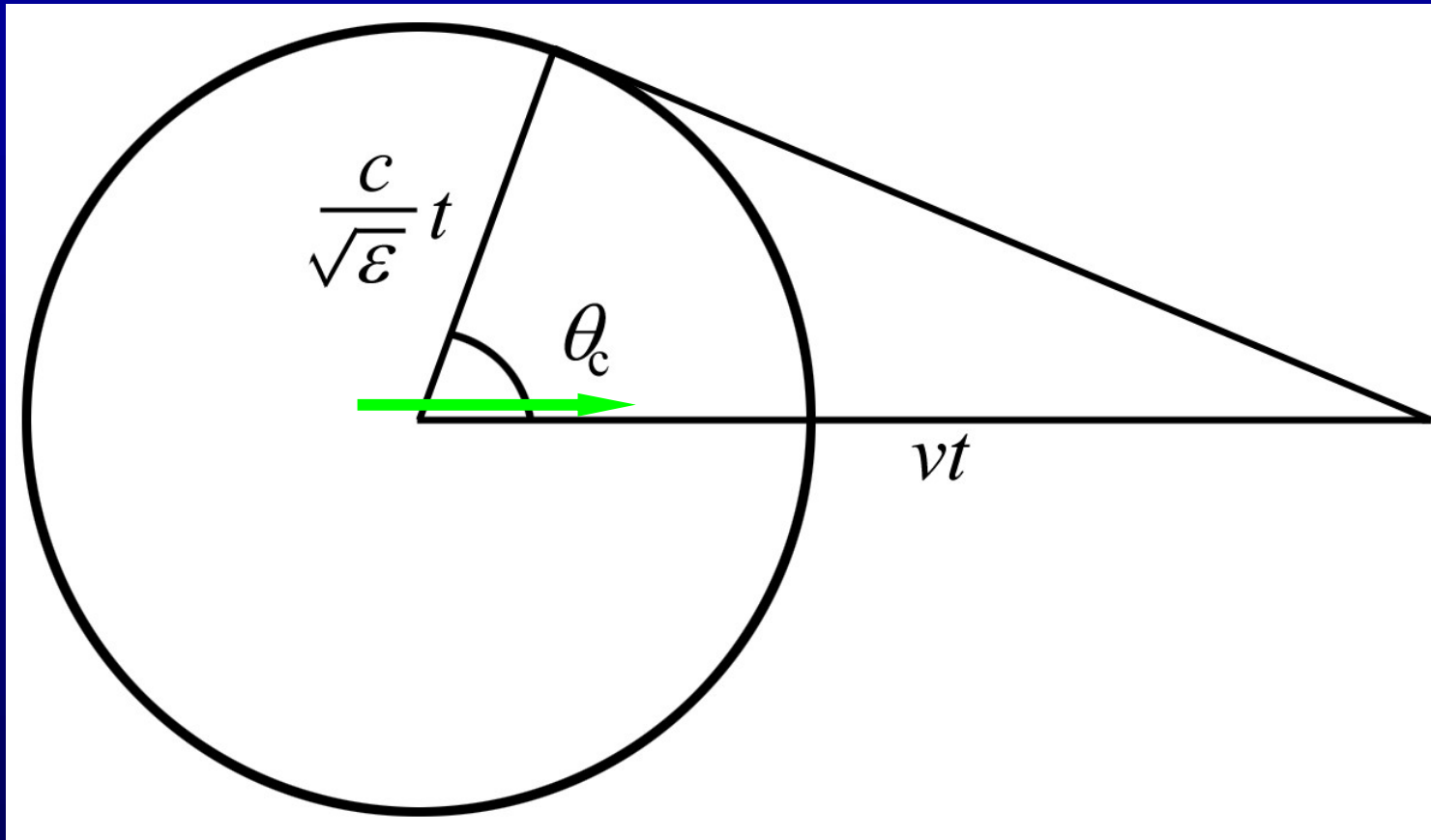


FA-18F Breaking Sound Barrier

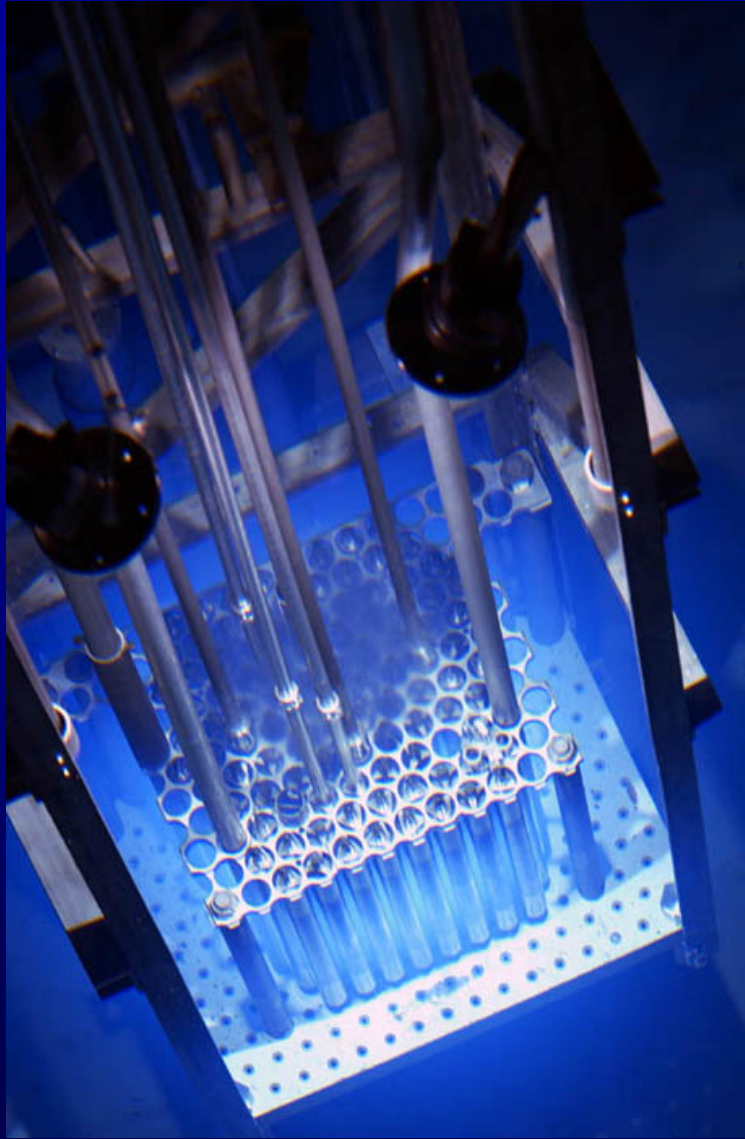
Prandtl-Glauert condensation cloud

N-wave

Cherenkov radiation



$$\cos \theta_c = \frac{c/\sqrt{\epsilon_r}}{v} = \frac{1}{\beta \sqrt{\epsilon_r}}, \beta = \frac{v}{c}$$



This view looks down on the fuel rods at Penn State's Breazeale Reactor. The reactor is a TRIGA model manufactured by General Atomics. The blue light surrounding the fuel is known as Cherenkov radiation, produced when charged particles travel through matter (in this case, water) at speeds greater than light.

Sources: the Penn State Radiation Science and Engineering Center

Optical Doppler effect (chapter 9)

$$\nu = \nu_0 \sqrt{\frac{1-\beta}{1+\beta}}, \quad \beta = \frac{u}{c} \quad \text{source and observer separating}$$

$$\nu = \nu_0 \sqrt{\frac{1+\beta}{1-\beta}}$$

Assignment: 8.17, 8.21

