

Part 1

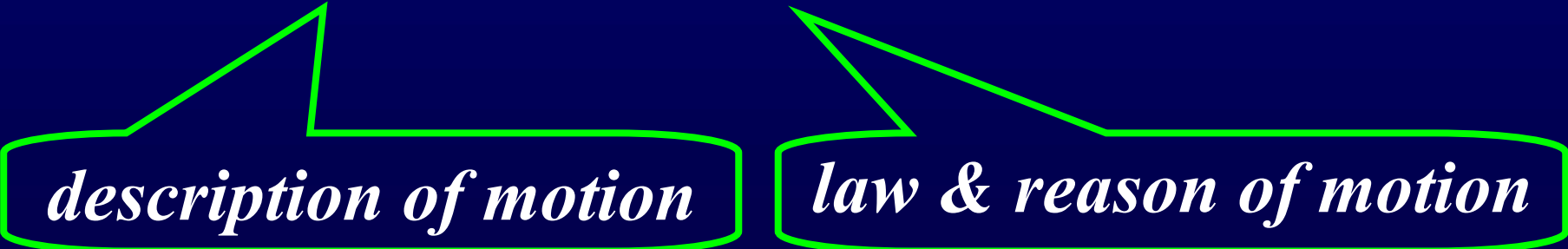
Mechanics

mechanical motion

- position (distance and direction)
- orientation
- deformation

spacetime

Kinematics & dynamics

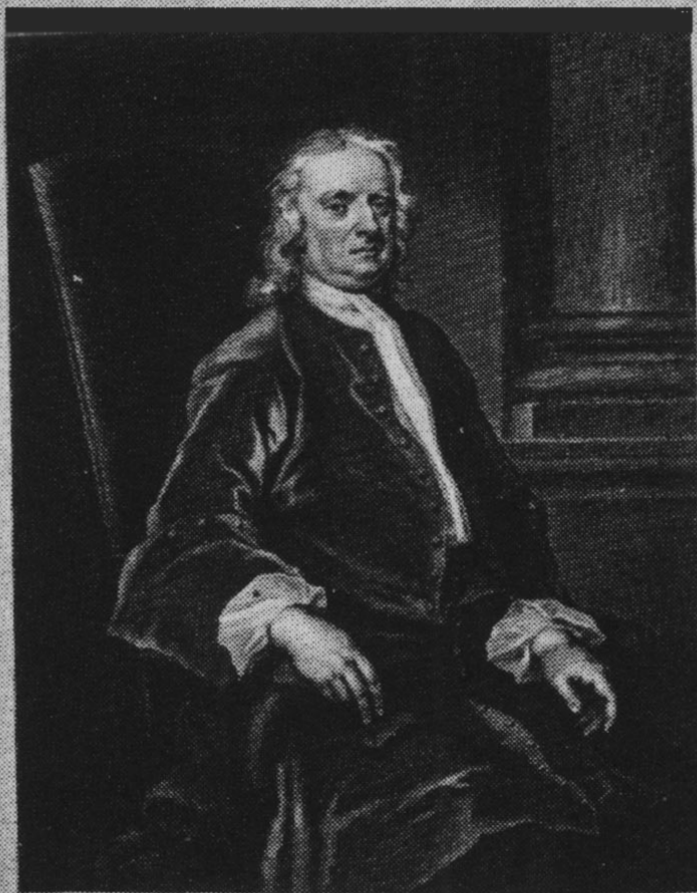


description of motion

law & reason of motion

Isaac Newton

1687 *Principia Methematica Philosophia
Naturalis*



ISAACUS NEWTON EQ. AUR. ÆT. 83.

J. Vanderbank pinxit 1703

Geo. Kneller Sculpsit 1704

PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

AUCTORE
ISAACO NEWTONO, EQ. AUR.

Editio tertia aucta & emendata.

LONDINI:
Apud GUIL. & JOH. INNYS, Regiæ Societatis typographos.
MDCCXXVI.

Isaac Newton

1687 *Principia Methematica Philosophia
Naturalis*

Albert Einstein

1905 *special theory of relativity*

1916 *general theory of relativity*

Heisenberg, Dirac, Schrödinger et al

1920s *Quantum mechanics*

Few-body and many-body

3-body problem (n-body problem)

- Oscar II, King of Sweden set a prize
- The prize was finally awarded to Henry Poincaré (who did not solve the original problems, but contributed a lot of important ideas that “inaugurate a new era in the history of celestial mechanics” (Karl Weierstrass)).
- Karl F. Sundman for $n=3$ in 1912 (*Proved solutions exist*)
- Qiudong Wang generalised to the case of $n > 3$ bodies by in the 1990s

Thermodynamic system 10^{23}

Macroscopic and microscopic

scale, size;

two levels

VS

abbr. for versus

deterministic vs uncertain

- quantum mechanical **uncertainty** of microscopic objects;
- statistical **uncertainty** of individual particle in many-particle system;
- **unpredictability** in nonlinear dynamic system. (deterministic unpredictability)

Chapter 2

Kinematics

2.1 Mechanical motion and moving object

translational motion — *particle or mass point*
position change **system of particles**
 continuum or fluid

rotational motion — rigid body orientation change

oscillation and deformation — elastic or plastic
relative position change bodies

Motion of the earth

<i>scale</i>	<i>motion</i>	<i>earth</i>
$1.5 \times 10^{11}\text{m}$	revolution (mean orbital radius)	mass point
$6.4 \times 10^6\text{m}$	rotation (mean radius)	rigid body
?	earthquake, crust movement	elastic or plastic body

Motion is relative.

Reference frame, or coordinate system is needed. Spacetime coordinates(Chapter 9)

2.2 Translation

position change

Δr displacement

Δs path

位移

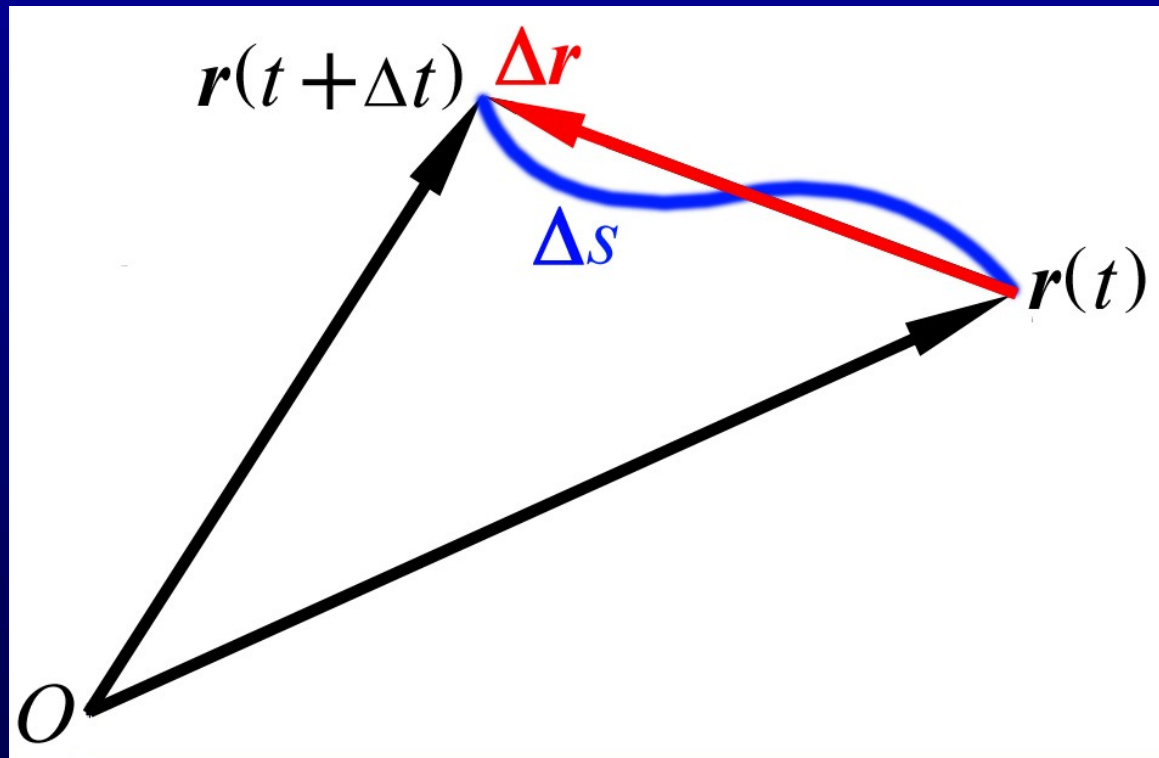
Average velocity

$$\bar{\mathbf{v}} = \frac{\Delta \mathbf{r}}{\Delta t}$$

Average speed

$$\bar{v} = \frac{\Delta s}{\Delta t}$$

速率



instantaneous velocity and speed

$$\mathbf{v} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \mathbf{r}}{\Delta t} = \frac{d\mathbf{r}}{dt}, \quad v = \lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{d\mathbf{r}}{ds} \frac{ds}{dt} = v \mathbf{e}_t \quad \bar{\mathbf{v}} \neq \bar{v} \mathbf{e}_t$$

acceleration

$$\mathbf{a} = \frac{d\mathbf{v}}{dt}$$

traditional kinematics:

$\Delta r(r)$, $v(v)$, and a

$r \rightarrow v \rightarrow a ; a \rightarrow v \rightarrow r$

$$\int a dt = v - v_0$$

$$\int v dt = r - r_0 \quad \int_0^t v dt = r - r_0$$

or

$$v = v_0 + \int a dt$$

$$r = r_0 + \int v dt$$

Example 2.1

$$\mathbf{v}_A = \mathbf{g}t$$

$$\mathbf{v} = vk$$

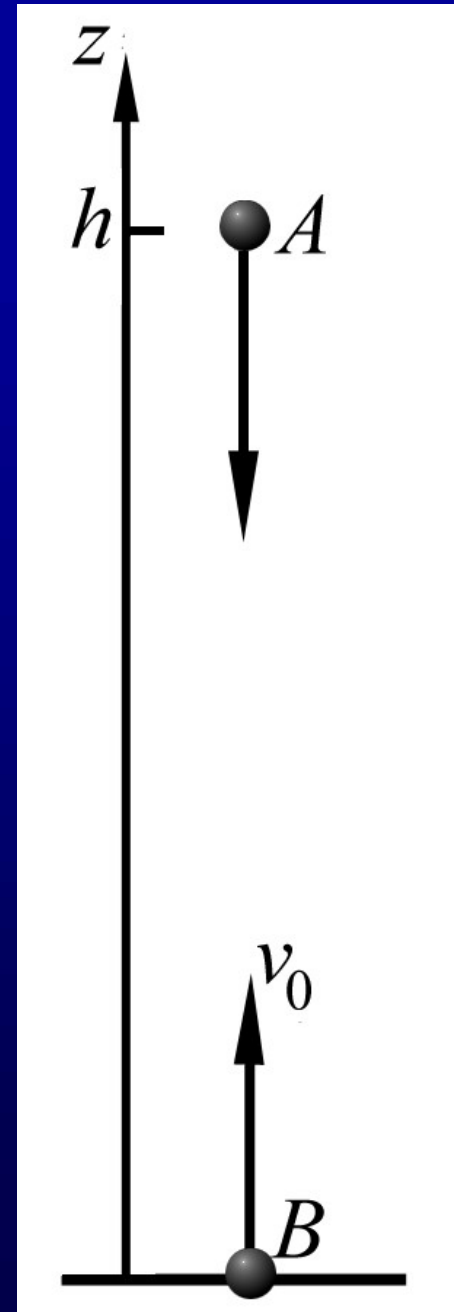
$$\mathbf{v}_B = \mathbf{v}_0 + \mathbf{g}t$$

$$\mathbf{g} = -g\mathbf{k}$$

$$\int_0^t \mathbf{g}t dt = -g\mathbf{k} \int_0^t t dt = -\frac{1}{2}gt^2\mathbf{k}$$

$$h_A = h - \frac{1}{2}gt^2$$

$$h_B = v_0 t - \frac{1}{2}gt^2$$



h_A, h_B are z-components of the **position vectors** rather than **paths**.

Collision takes place when $h_A = h_B$ and we have

$$t = h/v_0$$

Substituting it into the expression for h_A or h_B , we have

$$h_C = h \left[1 - \frac{gh}{2v_0^2} \right]$$

Discussion:

Assigned v_0, h satisfy $v_0^2 \geq \frac{1}{2}gh$

In case $v_0^2 \not\geq \frac{1}{2}gh$

Example 2.2 *Known* $a = -kv^2$

Find $v = v(z)$

Analysis: one-dimension (1D) implied
finite v , *finite* a . $\dim k$?

Solution: Write

$$a = \frac{dv}{dt} = \frac{dv}{dz} \frac{dz}{dt} = v \frac{dv}{dz} = -kv^2$$

we have

$$\frac{dv}{dz} = -kv$$

$$\int_{v_0}^v \frac{dv}{v} = -k \int_{z_0}^z dz$$

$$\ln \frac{v}{v_0} = -k(z - z_0)$$

$$v = v_0 \exp[-k(z - z_0)]$$

Discussion:

$$v = v(t)? \quad \dots$$

$$a = \frac{dv}{dt} = -kv^2$$

$$-\frac{dv}{v^2} = kdt$$

$$-\int_{v_0}^v \frac{dv}{v^2} = \int_0^t kdt$$

$$\frac{1}{v} - \frac{1}{v_0} = kt \quad v = \frac{v_0}{1 + v_0 kt}$$

From $v = v_0 \exp[-k(z - z_0)] = \frac{dz}{dt}$

$$dz \exp[k(z - z_0)] = v_0 dt$$

$$\exp[k(z - z_0)] - 1 = v_0 kt$$

Substituting back to the expression for v yields

$$v = \frac{v_0}{1 + v_0 kt}$$



Galileo Galilei (1564—1642)

independence of motion

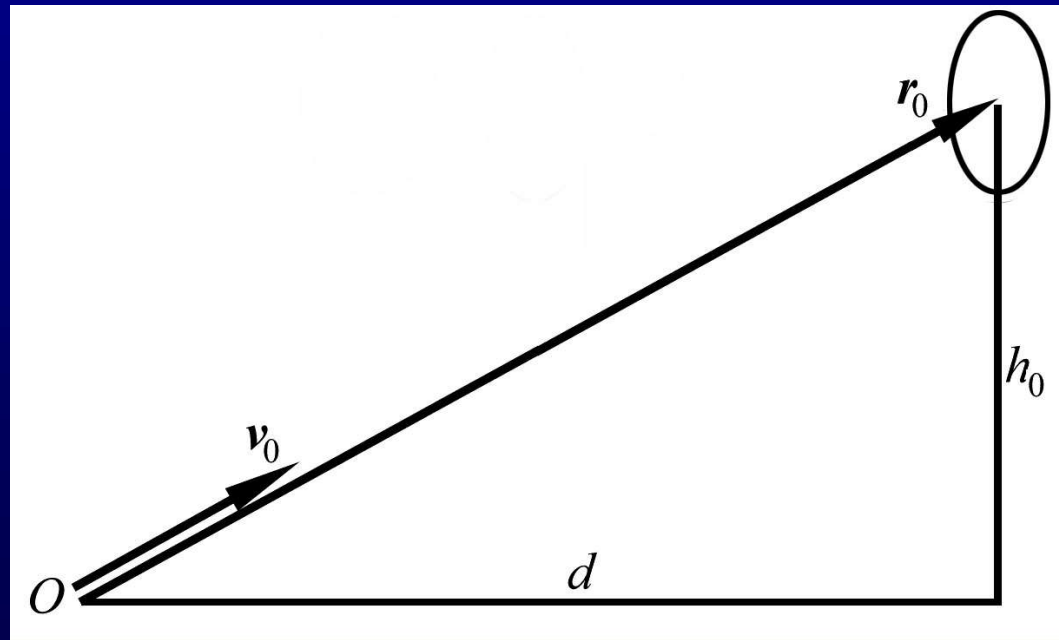
**composition and
decomposition of motion**

Example 2.3 Projectile

$$\mathbf{r}_c = \mathbf{r}_0 + \frac{1}{2} \mathbf{g} t^2$$

$$\mathbf{r}_b = \mathbf{v}_0 t + \frac{1}{2} \mathbf{g} t^2$$

clay pigeon



Hit-on condition:

$$r_c = r_b$$

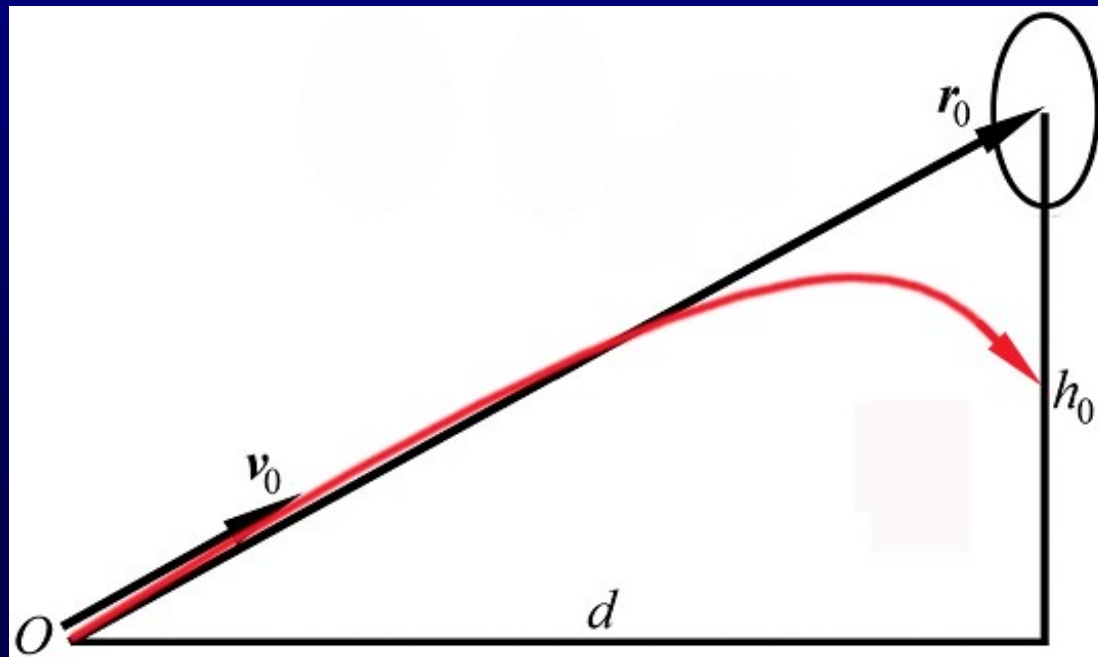
$$\longrightarrow r_0 = v_0 t$$

collinear (aim)

$$r_c = r_0 + \frac{1}{2} g t^2$$

$$r_b = v_0 t + \frac{1}{2} g t^2$$

Discussion: Write $r_0 = d i + h_0 k$



$$r_0^2 = d^2 + h_0^2 = v_0^2 t^2$$

$$\mathbf{r}_C = d\mathbf{i} + h_0\mathbf{k} - \frac{1}{2}g\mathbf{k}\frac{d^2 + h_0^2}{v_0^2}$$

$$\mathbf{r}_C \cdot \mathbf{k} = h_0 - \frac{1}{2}g\frac{d^2 + h_0^2}{v_0^2} \geq 0$$

$$v_0^2 \geq \frac{1}{2}gh_0\left(1 + \frac{d^2}{h_0^2}\right)$$

...

Circular motion

$$\rho = \rho e_\rho$$

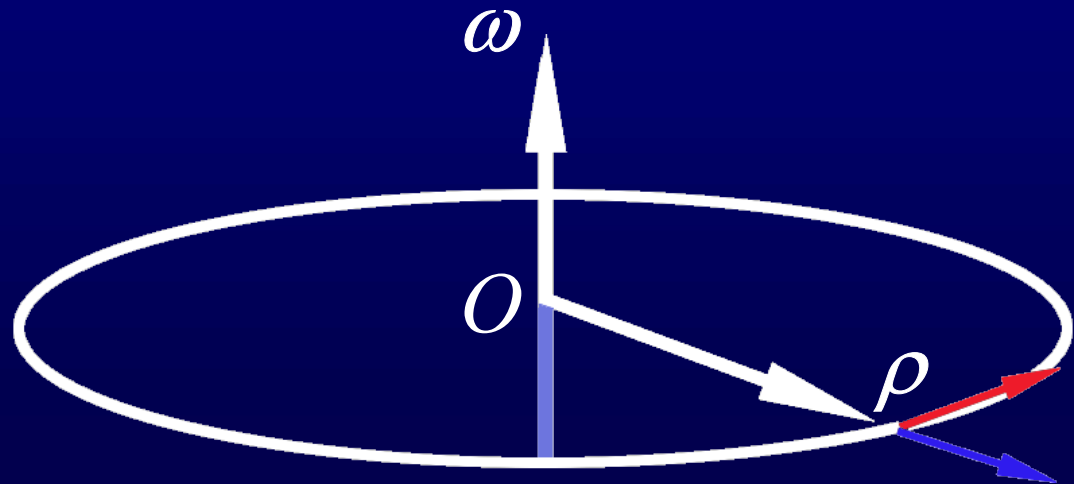
$$\dot{\rho} = \dot{\rho} e_\rho + \rho \dot{\phi} e_\phi \longrightarrow \rho \dot{\phi} e_\phi$$

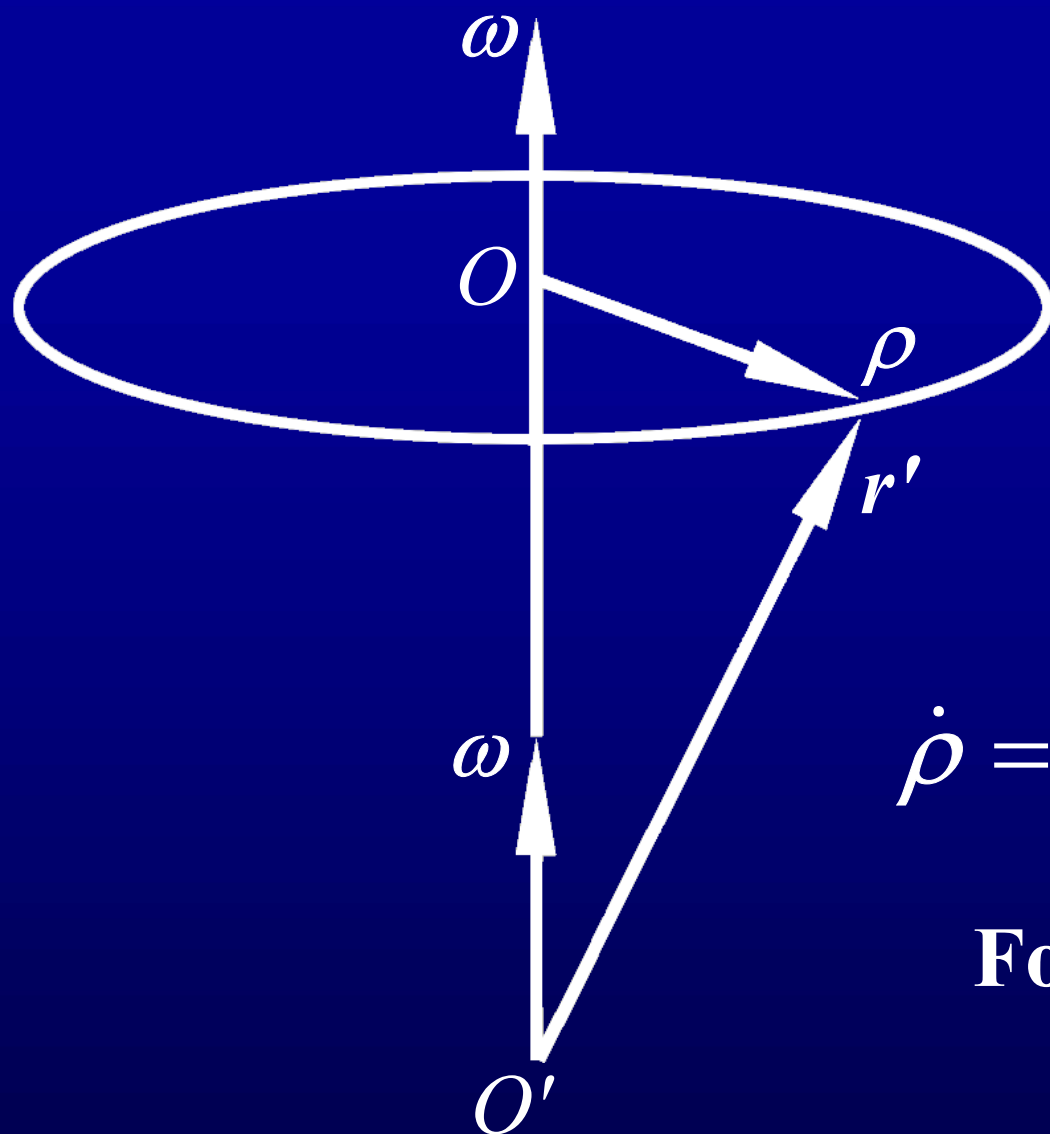
Define an **angular velocity** vector ω so that $\omega = \dot{\phi}$ and e_ρ, e_ϕ, ω are in the right-handed screw relation if $\dot{\phi} > 0$.

$$\dot{\rho} = \omega \times \rho$$

$$\rho \dot{\phi} e_\phi$$

$$\rho e_\rho$$





$$\rho = \overrightarrow{OO'} + r'$$

$$\therefore \dot{\rho} = \omega \times \rho$$

$$\dot{\rho} = \dot{r}'; \quad \omega \times \rho = \omega \times r'$$

For r' , we still have

$$\dot{r}' = \omega \times r'$$

radius vector

In **UCM** $\omega = \dot{\phi} = \text{constant}$

rev/s

$$\dot{\rho} = \omega \times \rho \quad \omega = \dot{\phi} = \frac{2\pi}{T} = 2\pi f$$

Acceleration: ($\omega = \text{const.}$)

$$\ddot{\rho} = \omega \times \dot{\rho} = \omega \times (\omega \times \rho)$$

$\longrightarrow -\omega^2 \rho$

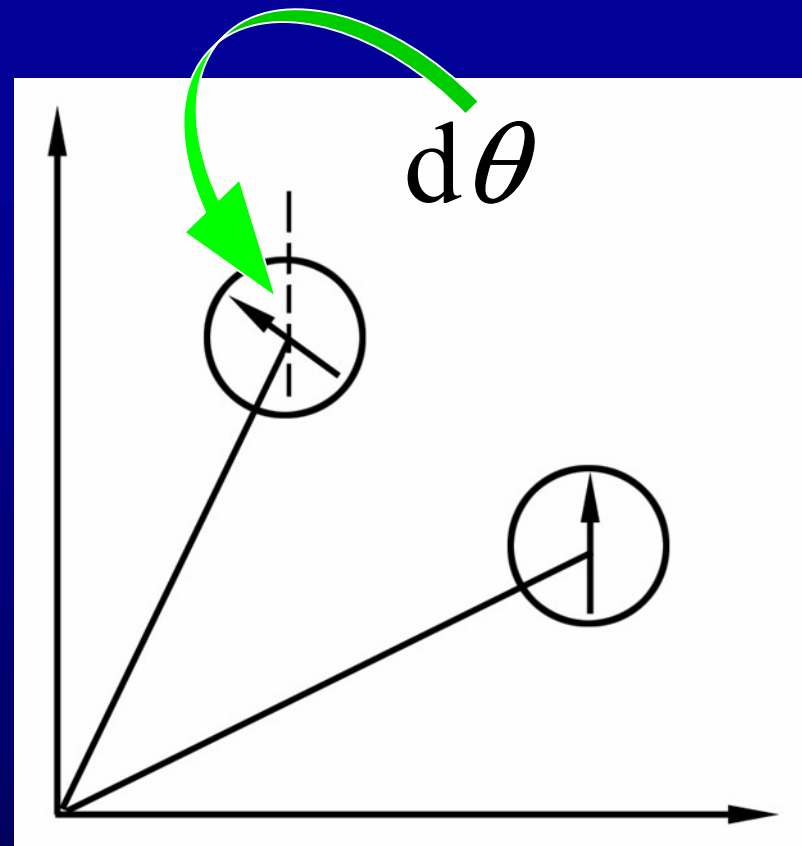
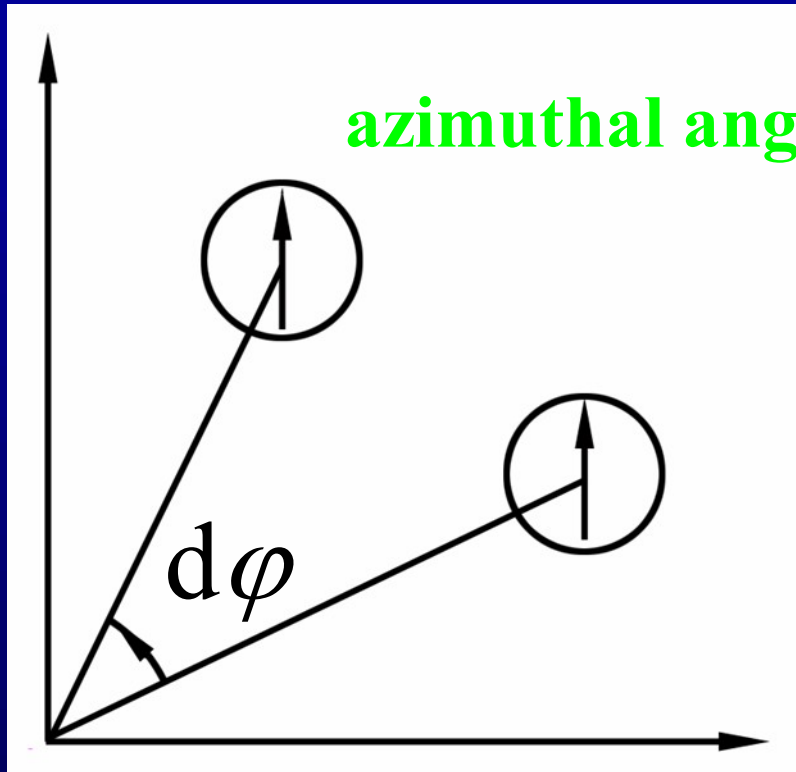
radian/s

Centripetal acceleration

向心加速度

$$A \times (B \times C) = B(A \cdot C) - C(A \cdot B)$$

2.3 Rotational motion



Translation

$\sim$ revolution $\omega = \frac{d\varphi}{dt}$

Translation

+rotation $\omega = \frac{d\theta}{dt}$



boomerang

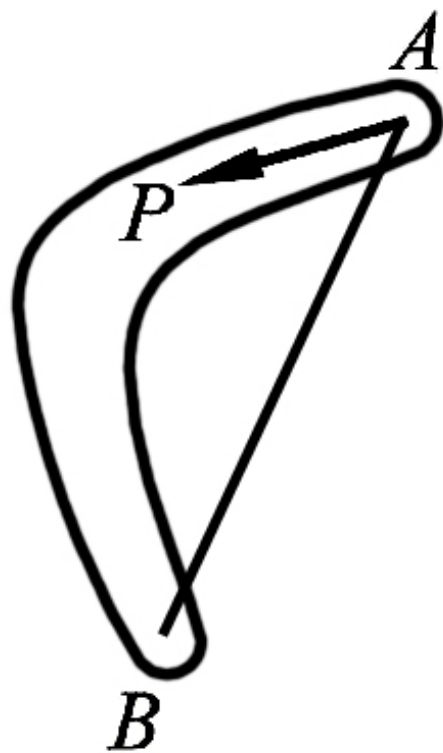
回旋镖，飞去来器



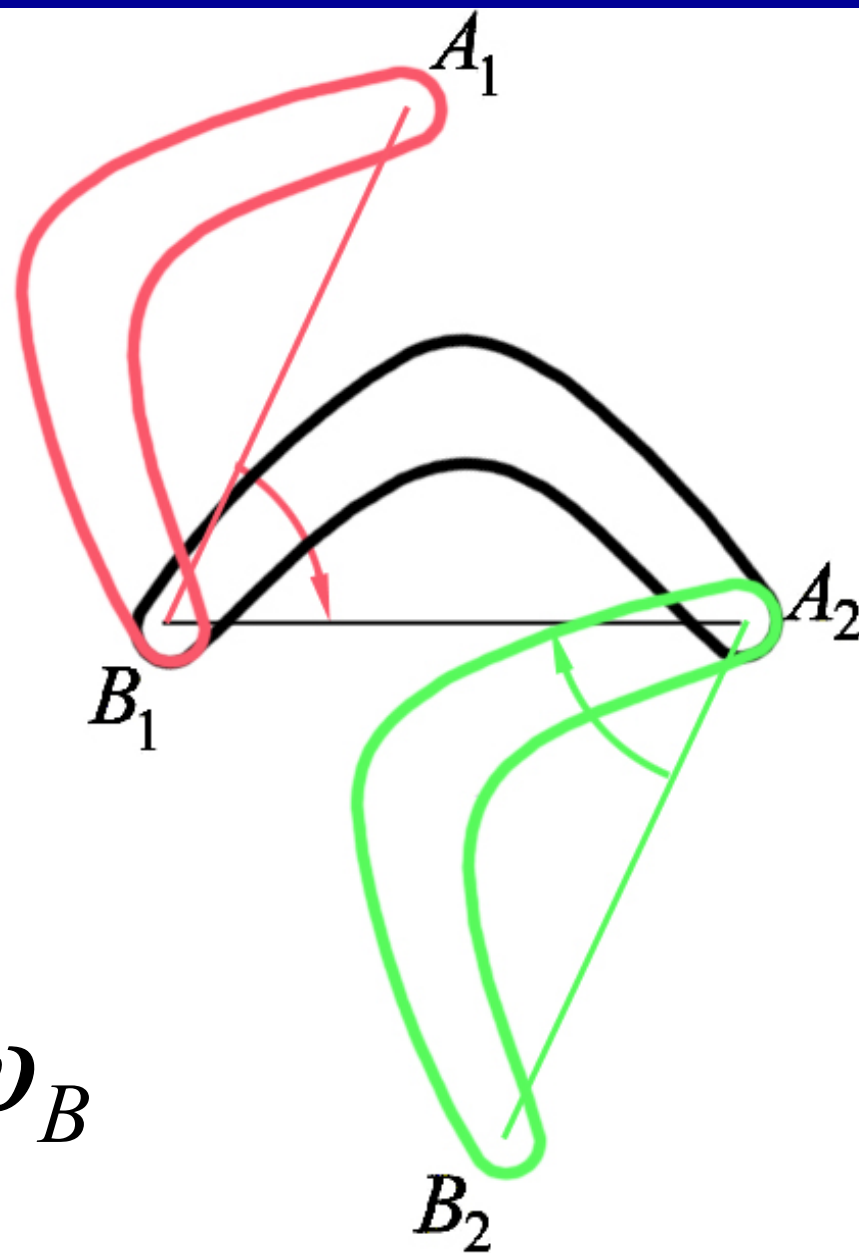
platypus: duck-billed animal
It is an oviparous mammal.

Monotremes(单孔类动物)are mammals that are oviparous, or egg-laying. There are only 3 extant species of monotremes: the platypus and two species of echidna (针鼹鼠).





$$\omega_A = \omega_B$$



$$\mathbf{v}_P = \mathbf{v}_A + \boldsymbol{\omega}_A \times \mathbf{r}_{AP}$$

$$\mathbf{v}_P = \mathbf{v}_B + \boldsymbol{\omega}_B \times \mathbf{r}_{BP} = \mathbf{v}_B + \boldsymbol{\omega}_B \times (\mathbf{r}_{BA} + \mathbf{r}_{AP})$$

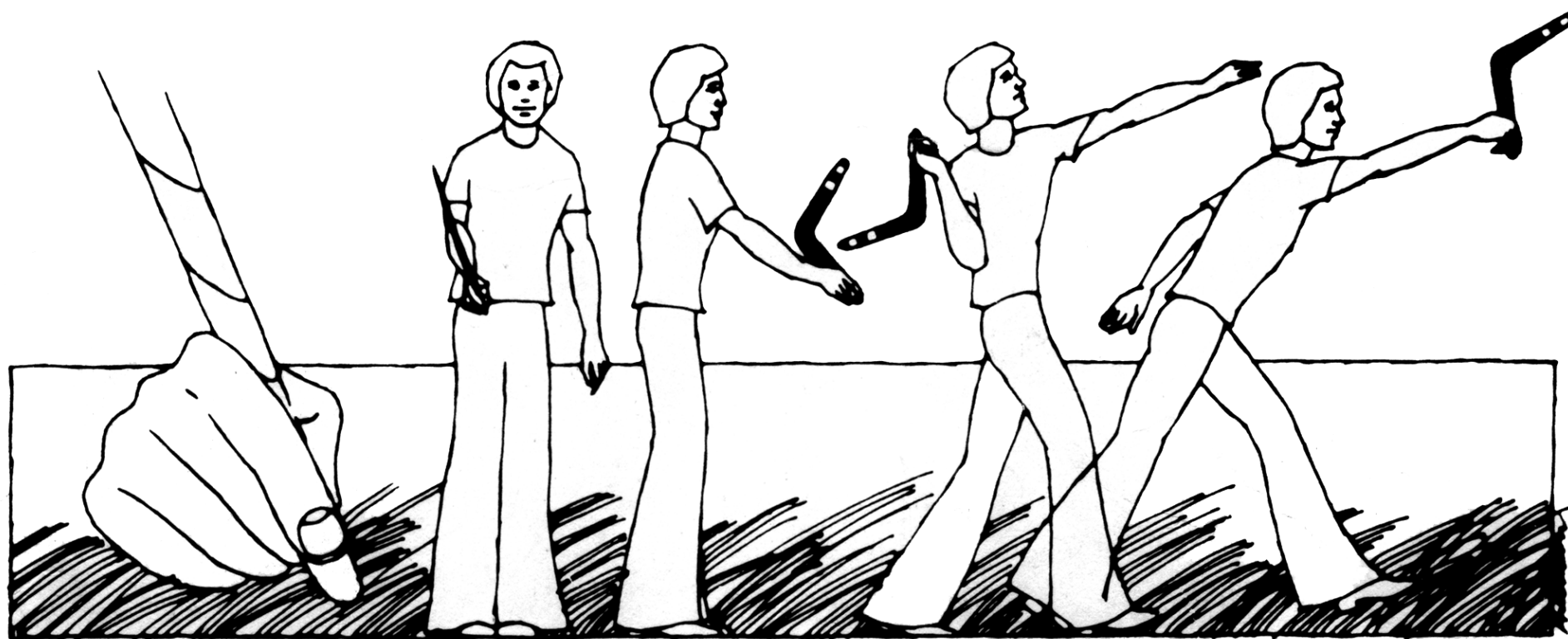
$$= \underline{\mathbf{v}_B} + \boldsymbol{\omega}_B \times \mathbf{r}_{BA} + \boldsymbol{\omega}_B \times \mathbf{r}_{AP}$$

$$= \underline{\mathbf{v}_A} + \boldsymbol{\omega}_B \times \mathbf{r}_{AP}$$

$$\Rightarrow \boldsymbol{\omega}_A \times \mathbf{r}_{AP} = \boldsymbol{\omega}_B \times \mathbf{r}_{AP}$$

$$\text{or } (\boldsymbol{\omega}_A - \boldsymbol{\omega}_B) \times \mathbf{r}_{AP} = 0$$

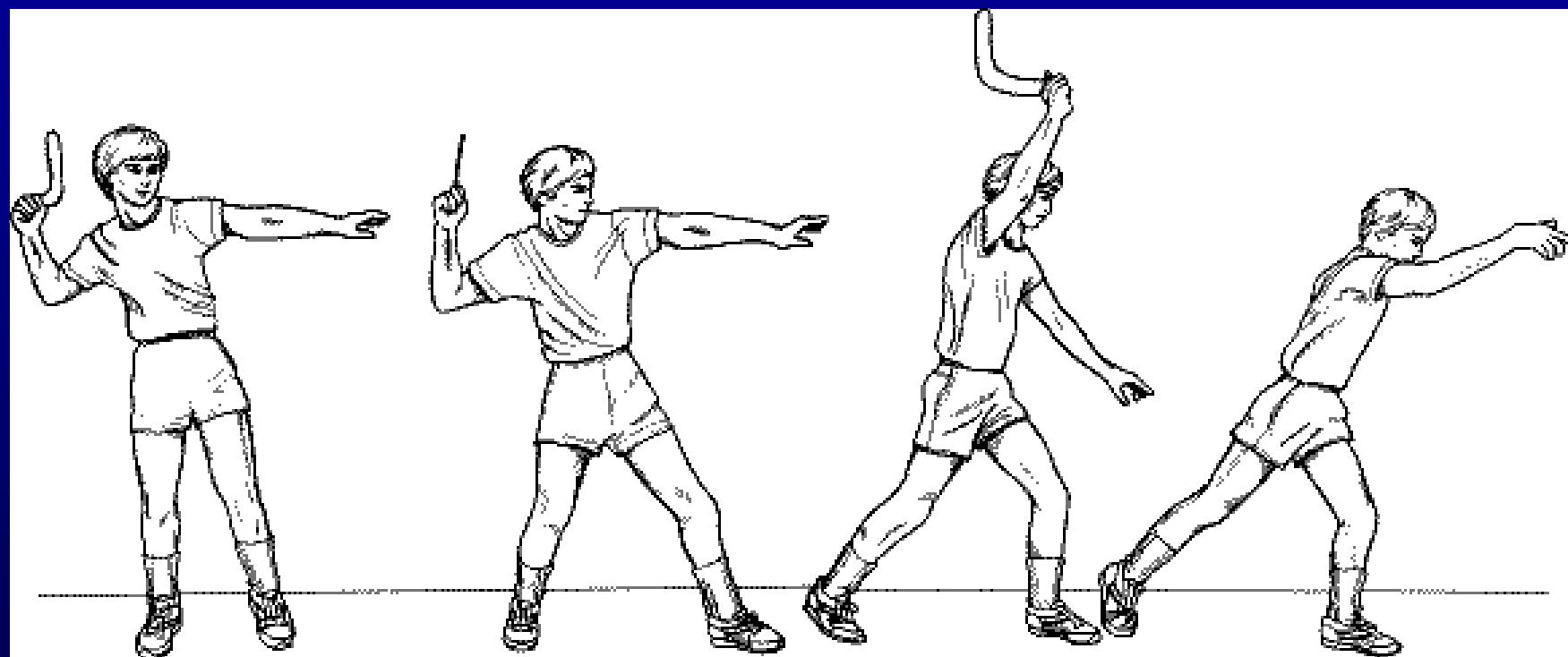
Since P is a arbitrary point, then $\boldsymbol{\omega}_A = \boldsymbol{\omega}_B$.



GRIP

STAND

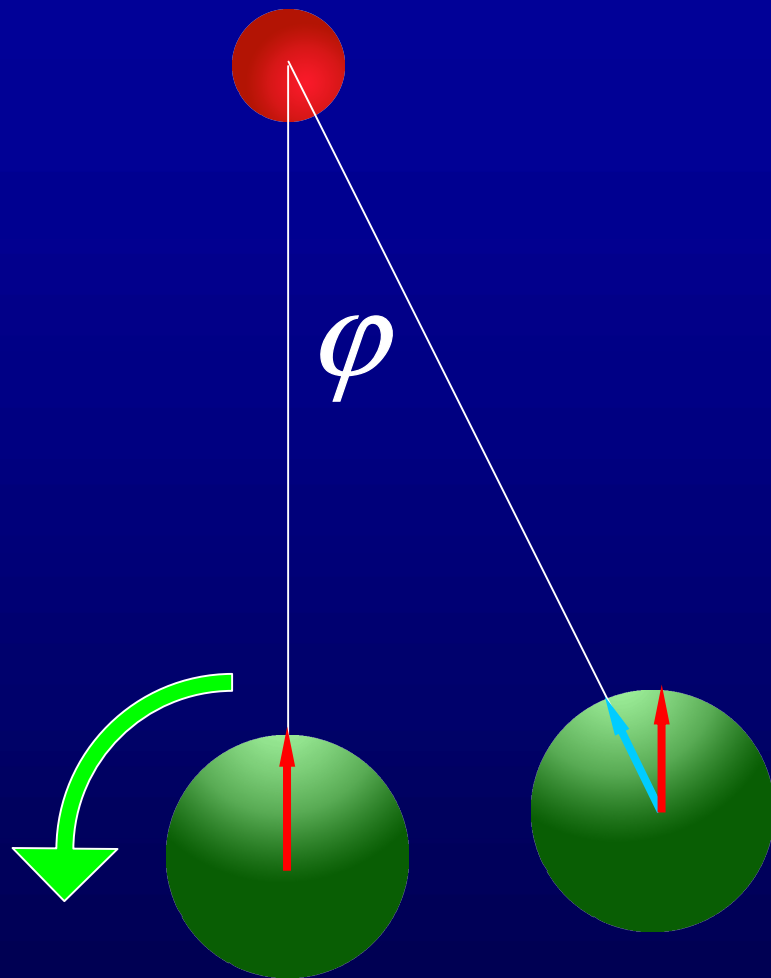
THROW



Example 2.4

The angular velocity of the **earth's** rotation

$$\omega = \frac{2\pi}{T}$$



● $T = 8.640 \times 10^4 \text{ s}$

● $T = 8.616 \times 10^4 \text{ s}$

$$\varphi \sim 2\pi/365$$

$$\Delta T = \varphi / \omega = 236 \text{ s}$$

$$\frac{2\pi}{T?}$$

● **sidereal day**

● **mean solar day**

$$\Delta T = \varphi / \omega$$

$$\omega = \frac{2\pi}{T - \Delta T}$$

$$\varphi \sim 2\pi/365$$

$$T = 8.640 \times 10^4 \text{ s}$$

mean solar day

$$\Delta T = \frac{T - \Delta T}{365}$$

$$\Delta T = \frac{T}{366} = 236 \text{ s}$$

Radiations from space

The origin of radio astronomy In 1931, Karl Jansky, an American engineer at the Bell Telephone Laboratories (now Bell Laboratories, part of Lucent Technologies). in New Jersey, studied static that was interfering with short-wave communication systems. He found that the static appeared four minutes earlier each day. Jansky knew that the stars rise four minutes earlier daily, and so he concluded that the static must be coming from beyond the solar system. He was actually receiving radio waves from the center of our galaxy.

Assignment: 2.3, 2.5, 2.7/2.8

2.4 Oscillation

periodic motion

$$A(t + T) = A(t)$$

period

frequency

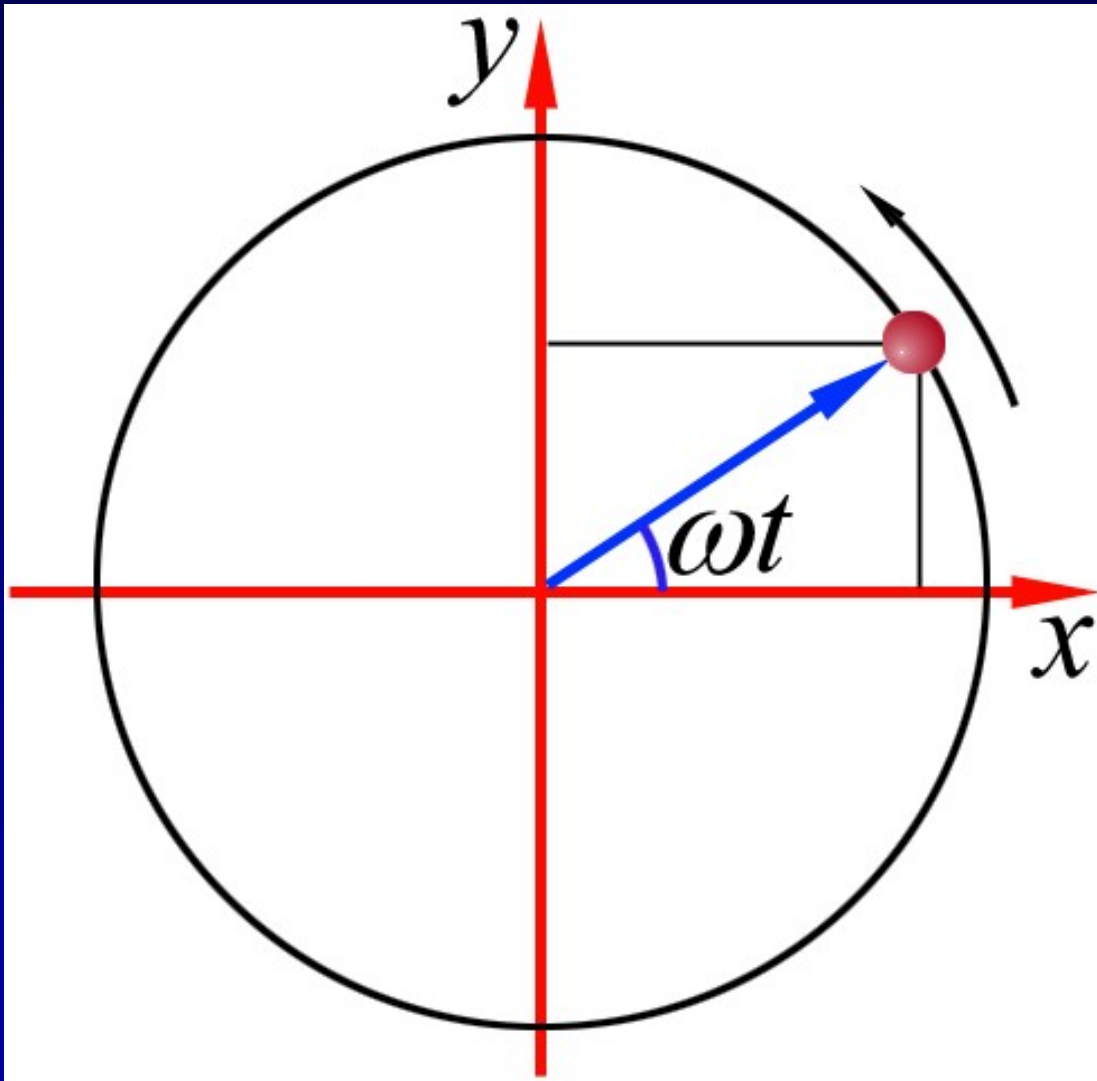
$$1/T \equiv f$$

- orbital motion of planets
- blood circulation
- ecological cycle
- economic cycle 繁荣/停滞/衰退/萧条/恢复
boom —stagnation—
—recession/depression—recovery—
-

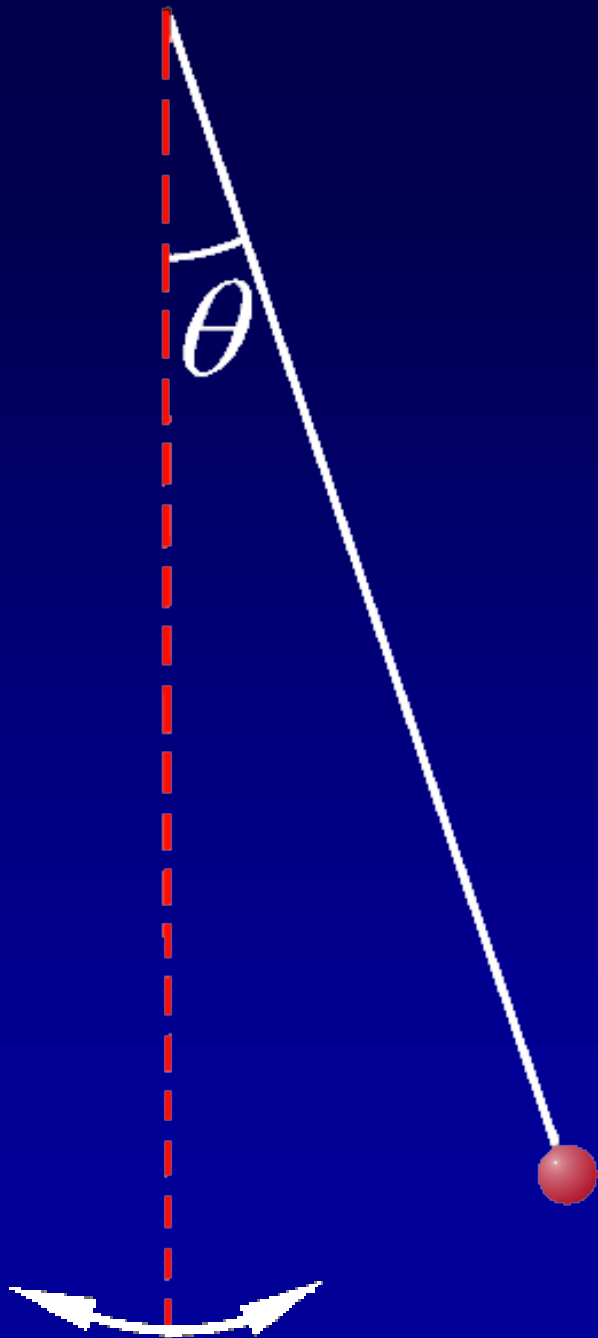
**"Recession is when your neighbor loses his job.
Depression is when you lose yours. And
recovery is when Jimmy Carter loses his."**

—Ronald Reagan in his presidential campaign

Spatially back and forth



$$\begin{cases} x = R \cos \omega t \\ y = R \sin \omega t \end{cases}$$



$$\theta = \theta_0 \cos \omega t$$

$$\dot{\theta} = -\omega \theta_0 \sin \omega t$$

$$x = A \sin \omega t + B \cos \omega t$$

$$\underline{x = A \cos(\omega t + \varphi)}$$

$$x = A e^{i\omega t} + B e^{-i\omega t}$$

A — Amplitude 振幅

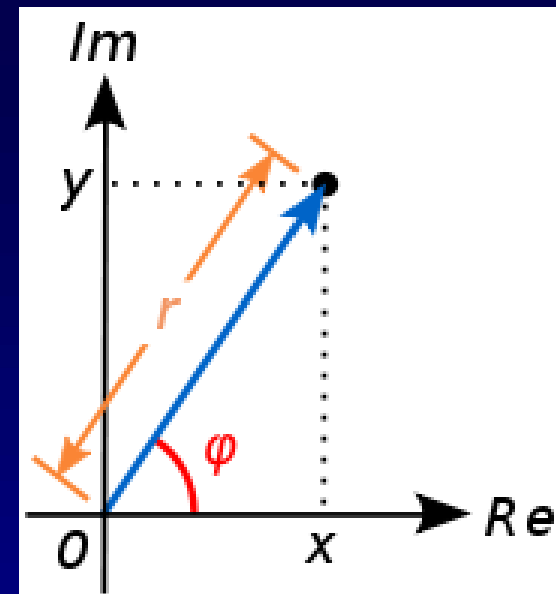
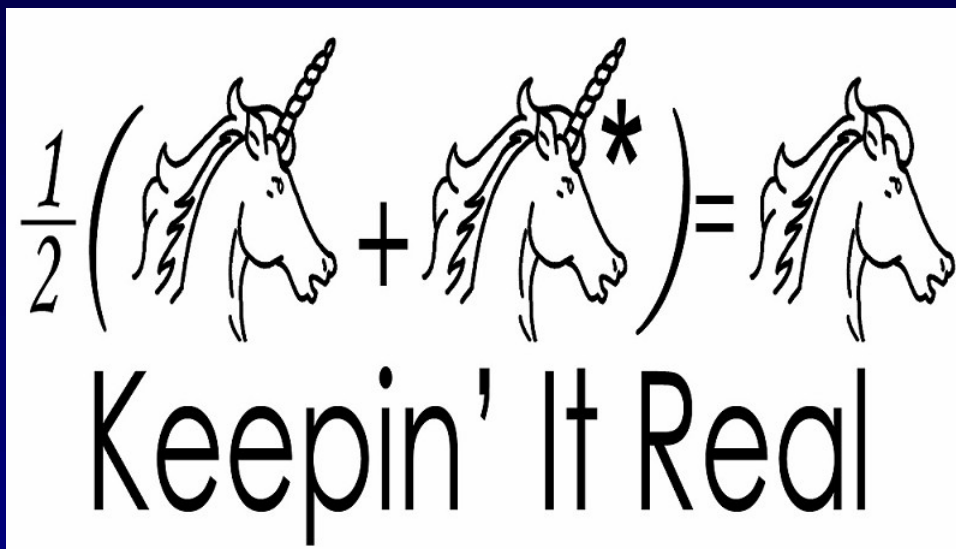
$\Phi \equiv \omega t + \varphi$ — Phase 相位

φ — Initial phase 初相位

ω — Angular (Circular) frequency 角频率

$$\Phi(T + t) - \Phi(t) = 2\pi \longrightarrow T = 2\pi/\omega$$

$$x = \tilde{A}e^{i\omega t}, \quad \tilde{A} = Ae^{i\varphi}, \quad A \text{ is real.} \quad \operatorname{Re} x = A \cos(\omega t + \varphi)$$



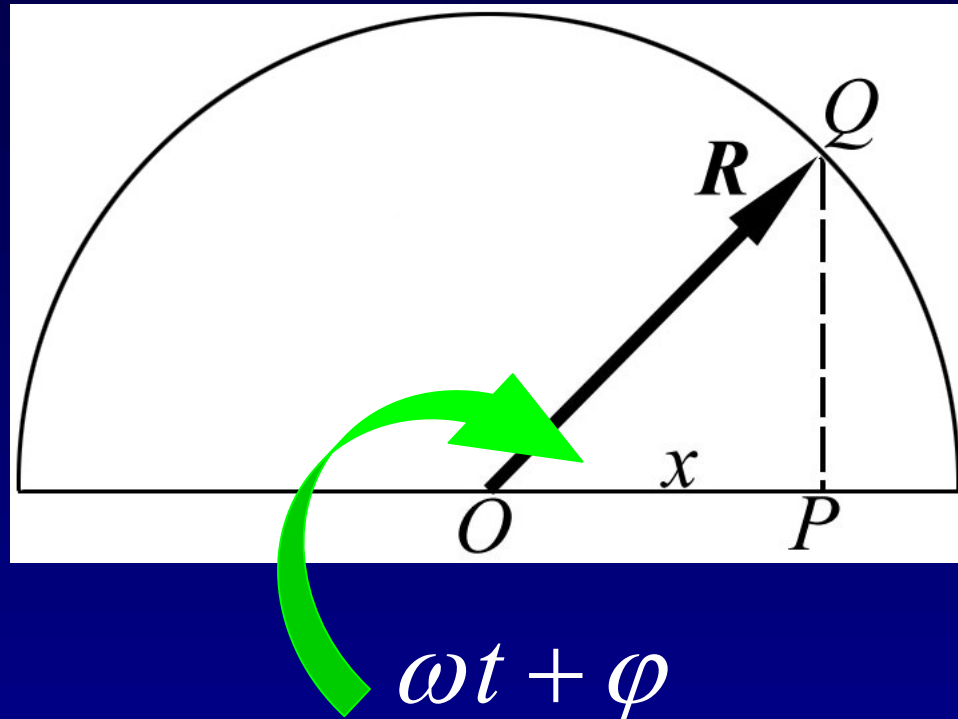
$z = x + iy$, $x = \operatorname{Re} z$, $y = \operatorname{Im} z$. real and imaginary parts.

$z^* = x - iy$ complex conjugate (复共轭) of z .

$z = re^{i\varphi}$, $r = |z| = \sqrt{x^2 + y^2}$ is modulus of z ;

$\varphi = \arg(z)$ is the argument (or phase) of z .

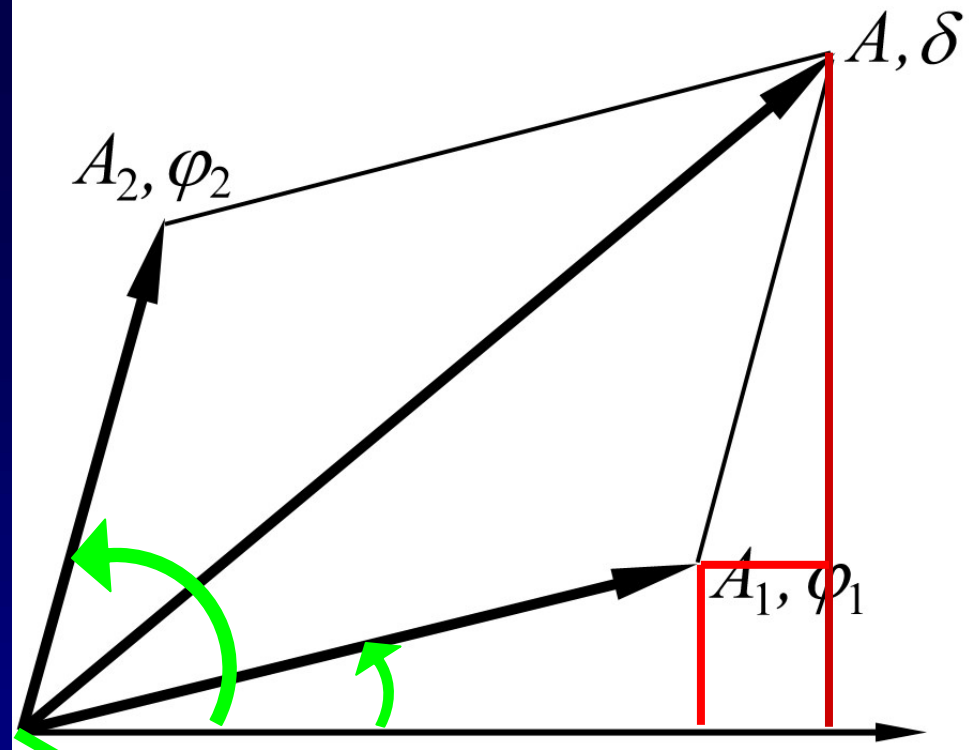
$e^{i\varphi} = \cos \varphi + i \sin \varphi$ – Euler's formula. $e^{i\pi} = -1$.



$$\begin{aligned} x &= R \cos(\omega t + \varphi) \\ &= \mathbf{R} \cdot \mathbf{i} \end{aligned}$$

rotating vector

Superposition of two oscillation



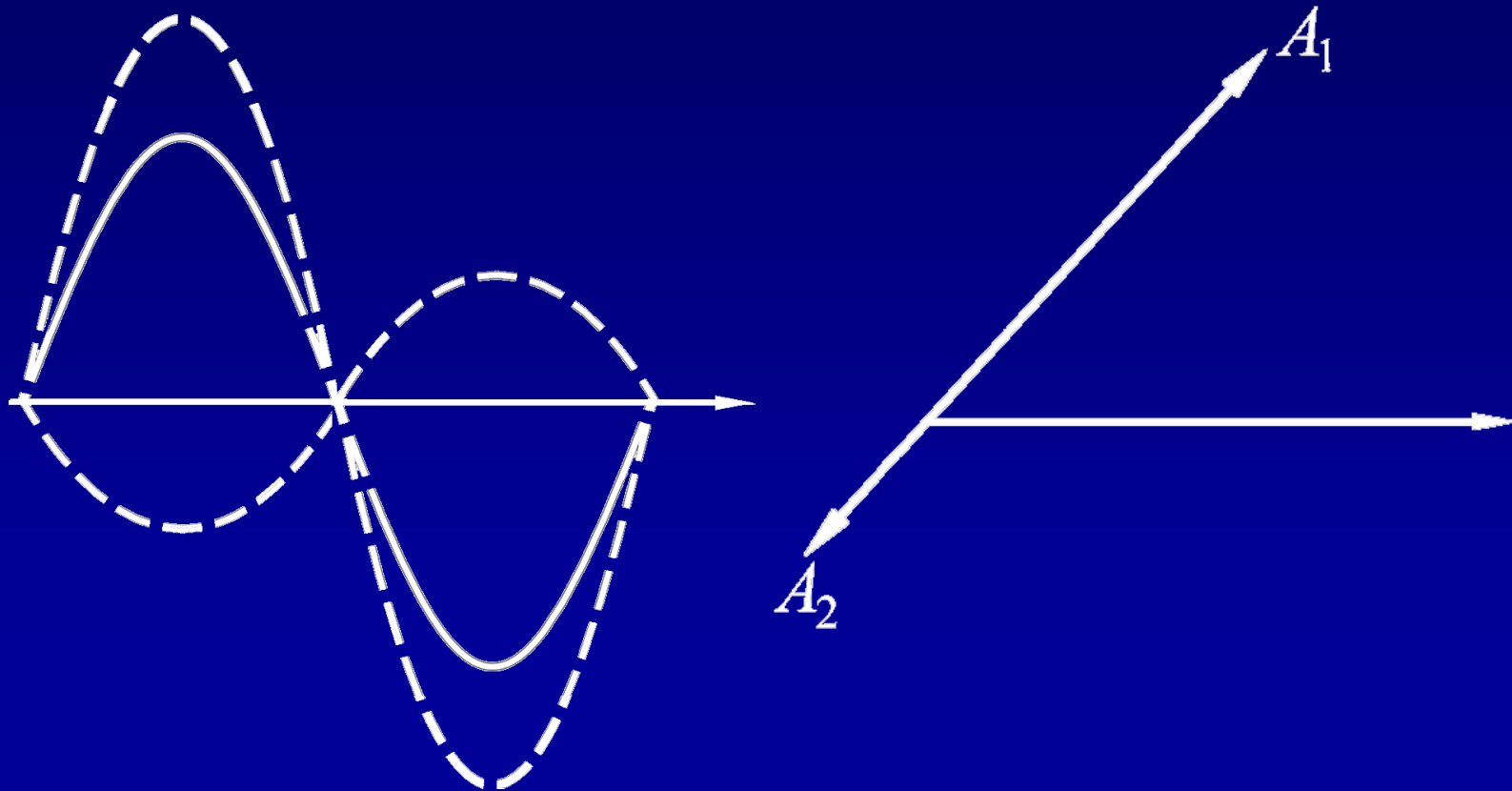
$$A_1 \cos(\omega t + \varphi_1) + A_2 \cos(\omega t + \varphi_2) = A \cos(\omega t + \delta)$$

$$A^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos(\varphi_2 - \varphi_1)$$

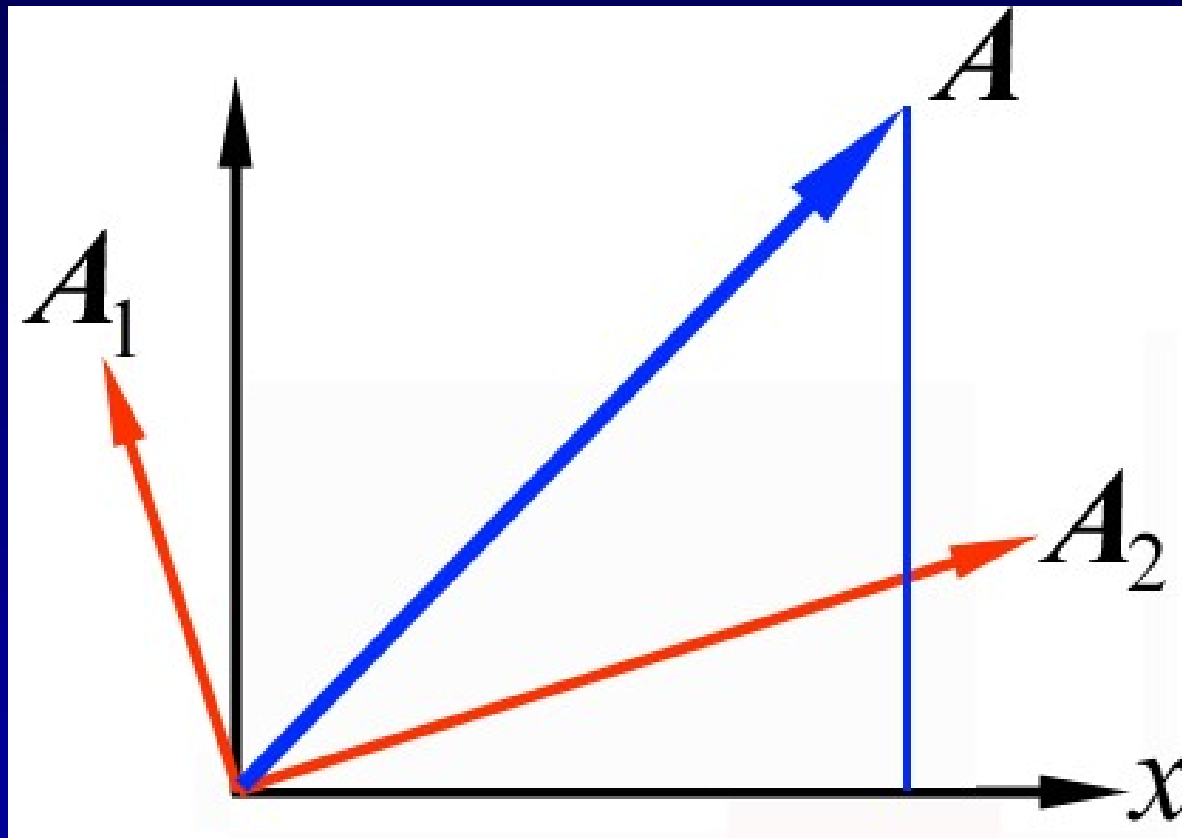
$$\tan \delta = \frac{A_1 \sin \varphi_1 + A_2 \sin \varphi_2}{A_1 \cos \varphi_1 + A_2 \cos \varphi_2}$$

$\varphi_1 = \varphi_2$ A_1, A_2 collinear. constructive

$\varphi_1 - \varphi_2 = \pi$ $A = |A_1 - A_2|$ destructive



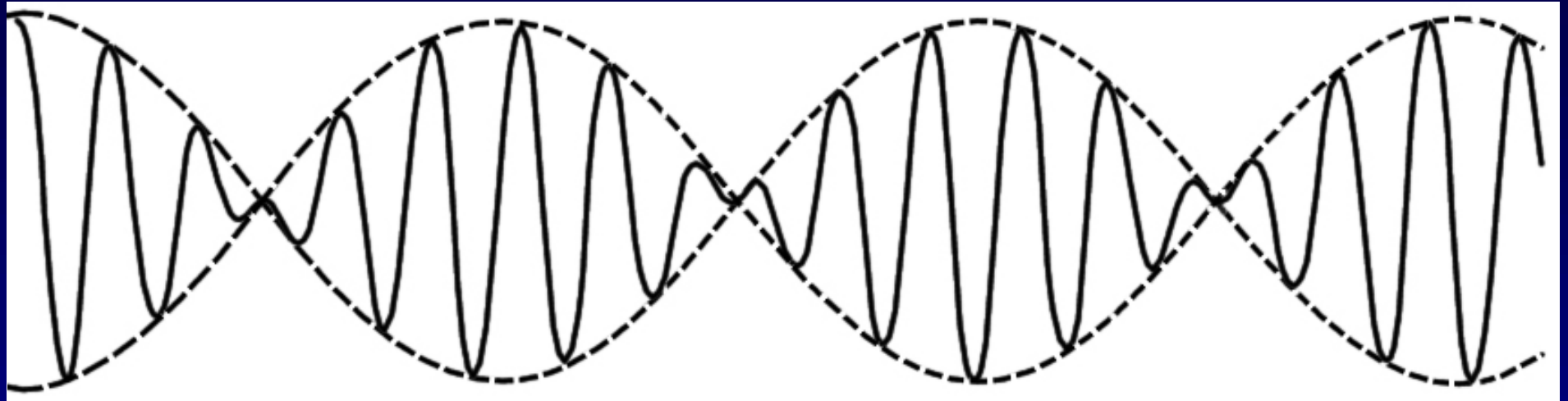
$$\varphi_1 - \varphi_2 = \pi/2 \quad A = \sqrt{A_1^2 + A_2^2} \quad \text{quadrature}$$



Oscillations of **different** frequencies

$$\begin{cases} x_1 = A_1 \cos(\omega_1 t) \\ x_2 = A_2 \cos(\omega_2 t) \end{cases} \quad \text{Set} \quad \begin{matrix} A_1 = A_2 = A \\ \varphi_1 = \varphi_2 = 0 \end{matrix} \quad ?$$

$$\begin{aligned} x_1 + x_2 &= A(\cos \omega_1 t + \cos \omega_2 t) \\ &= 2A \cos\left(\frac{\omega_1 - \omega_2}{2} t\right) \cos\left(\frac{\omega_1 + \omega_2}{2} t\right) \end{aligned}$$



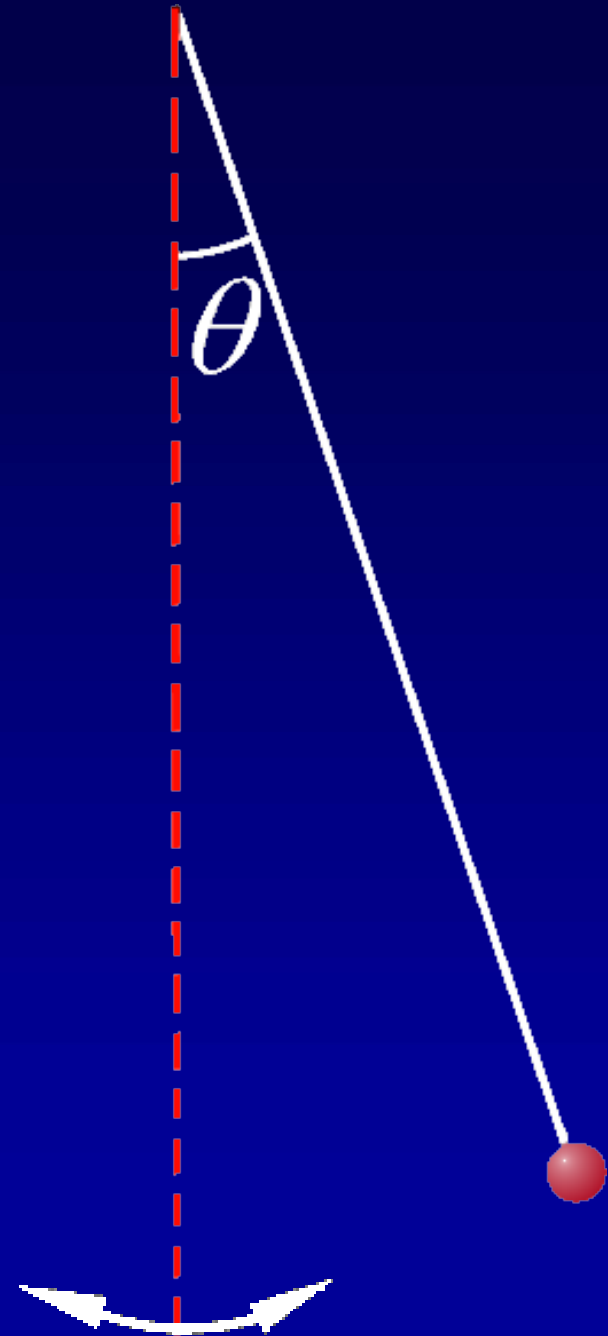
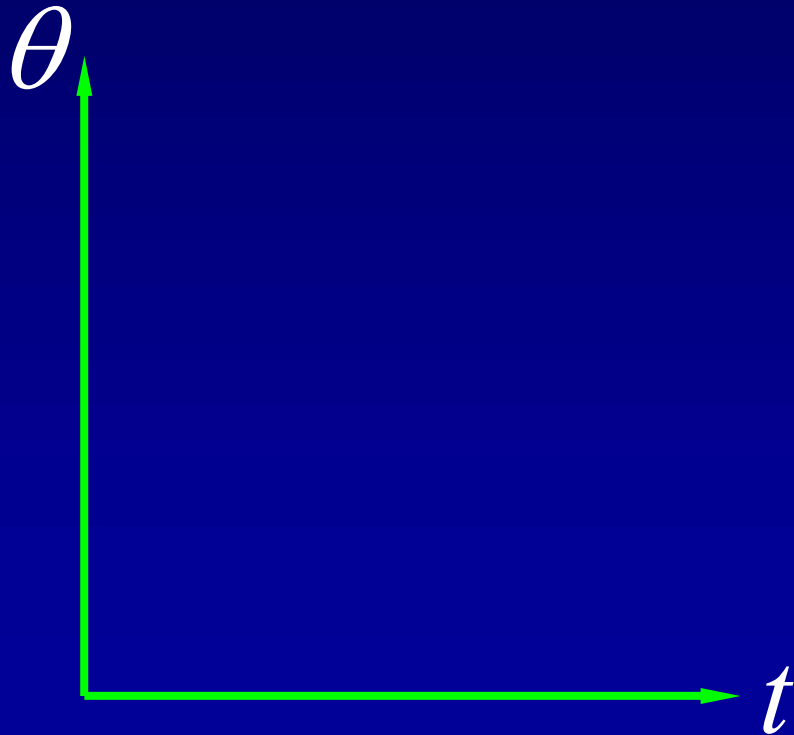
amplitude modulation

**Superposition of two perpendicular SHMs
yields Lissajous figures. Google!**

2.5 Phase space

$$\theta = \theta_0 \cos \omega t$$

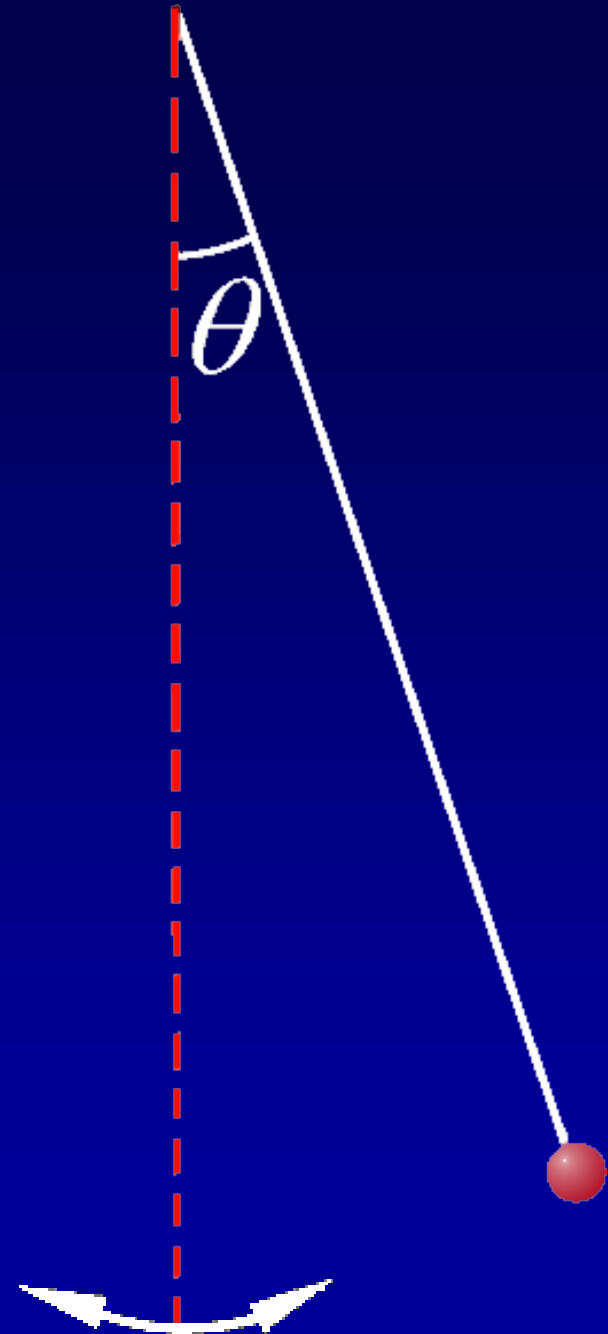
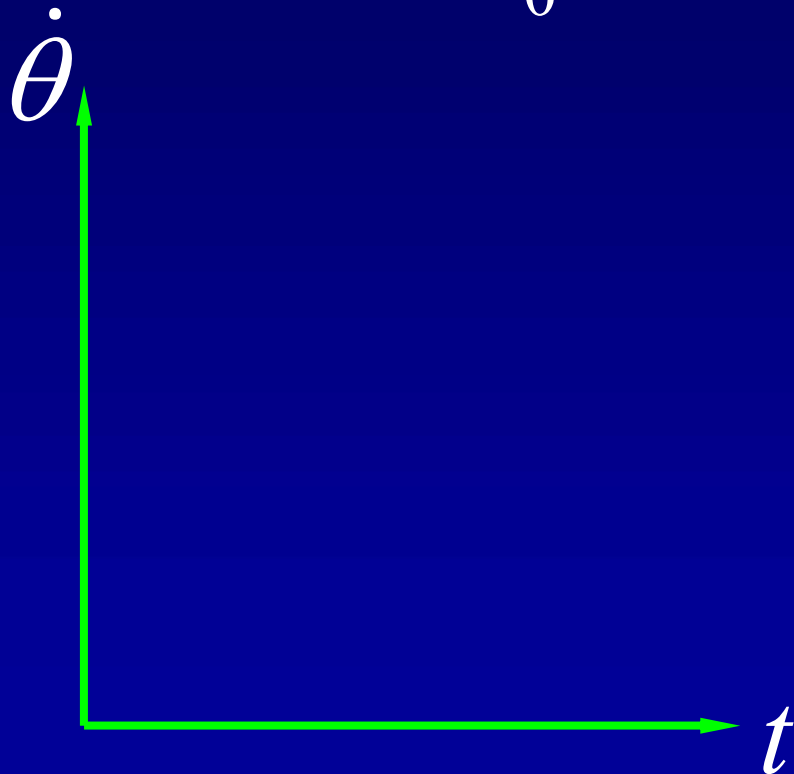
$$\dot{\theta} = -\omega \theta_0 \sin \omega t$$



2.5 Phase space

$$\theta = \theta_0 \cos \omega t$$

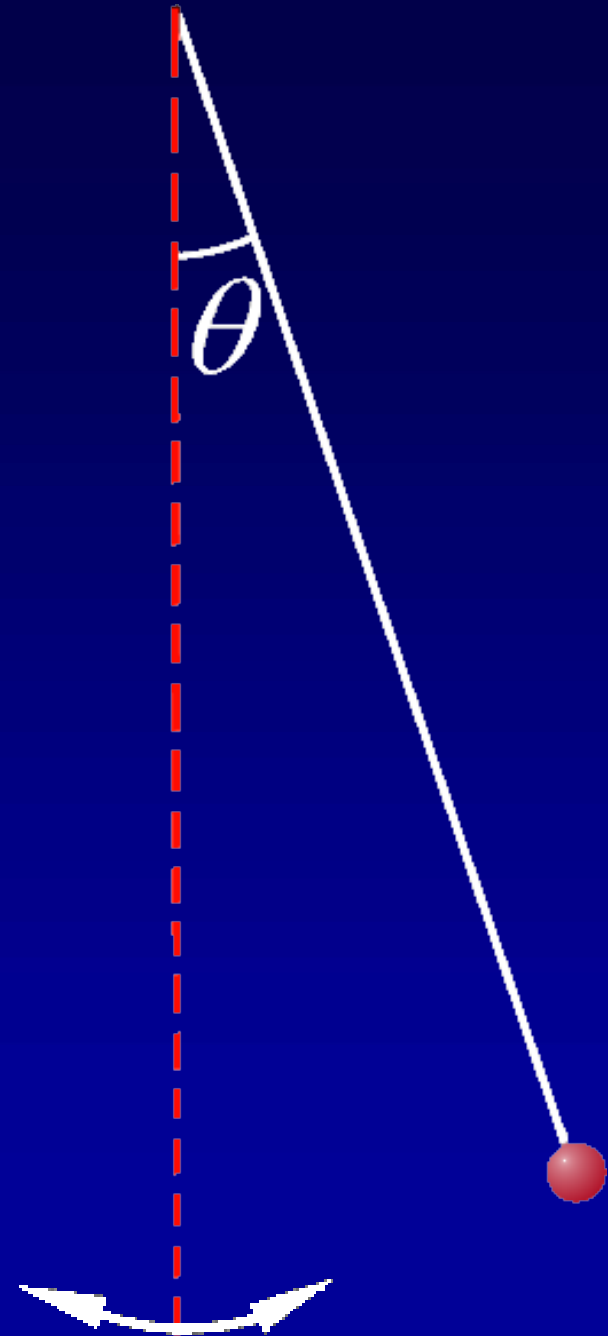
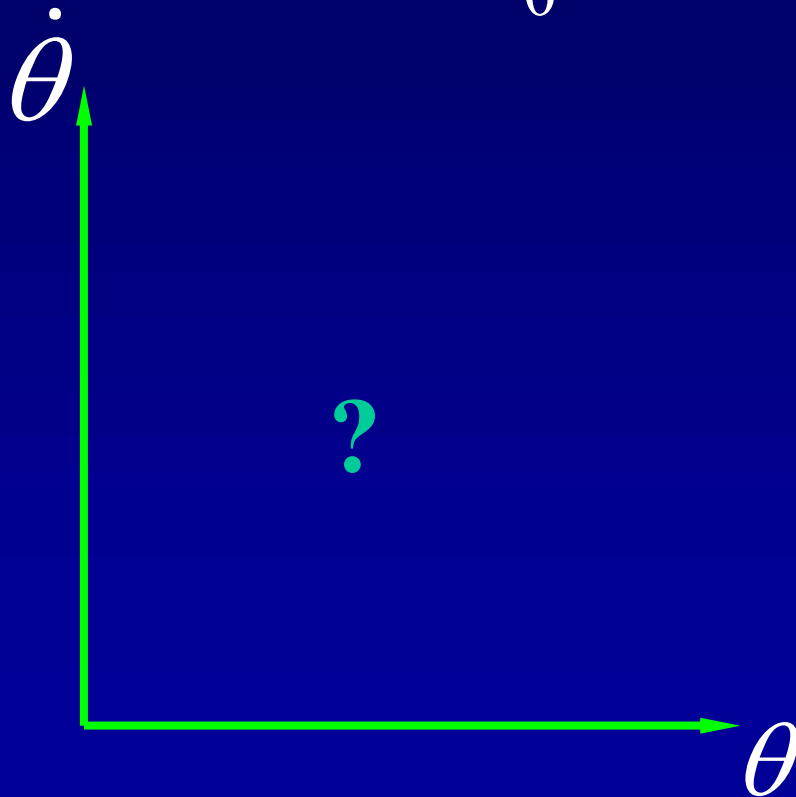
$$\dot{\theta} = -\omega \theta_0 \sin \omega t$$



2.5 Phase space

$$\theta = \theta_0 \cos \omega t$$

$$\dot{\theta} = -\omega \theta_0 \sin \omega t$$

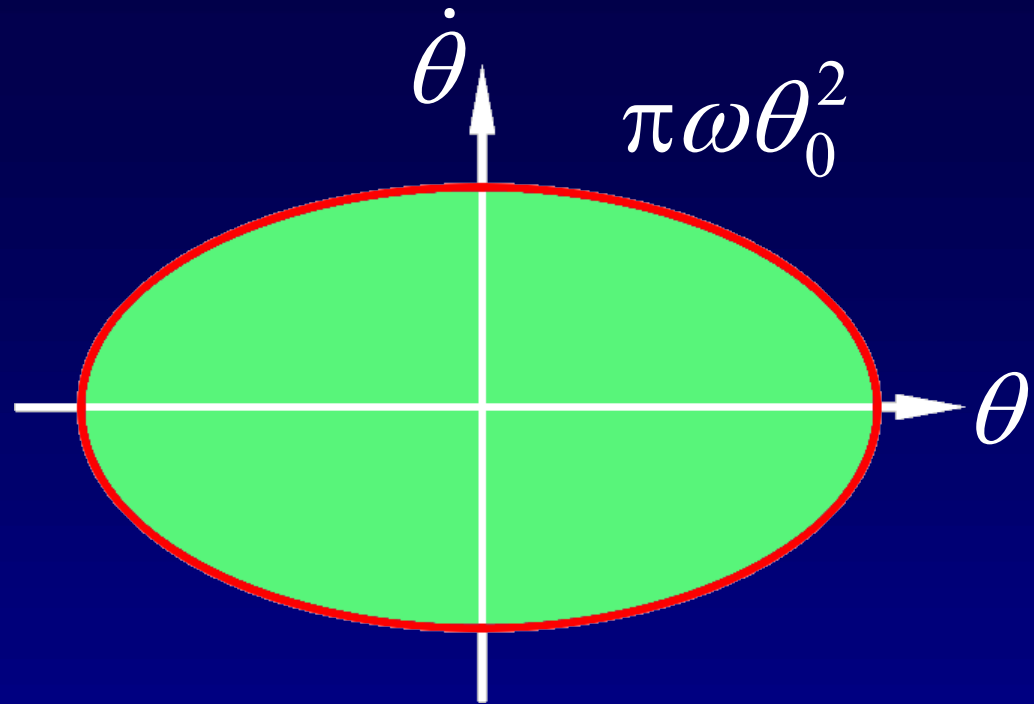


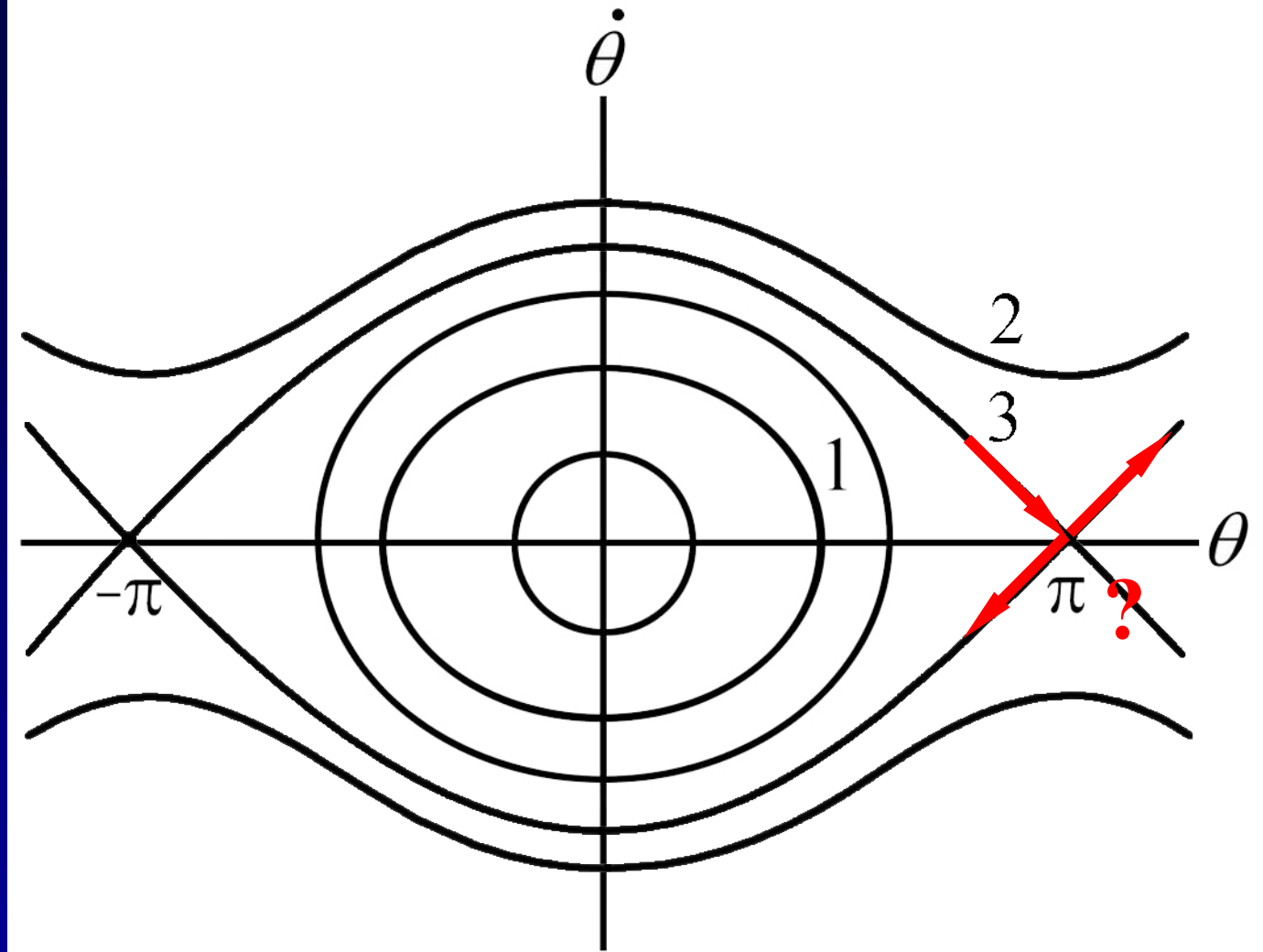
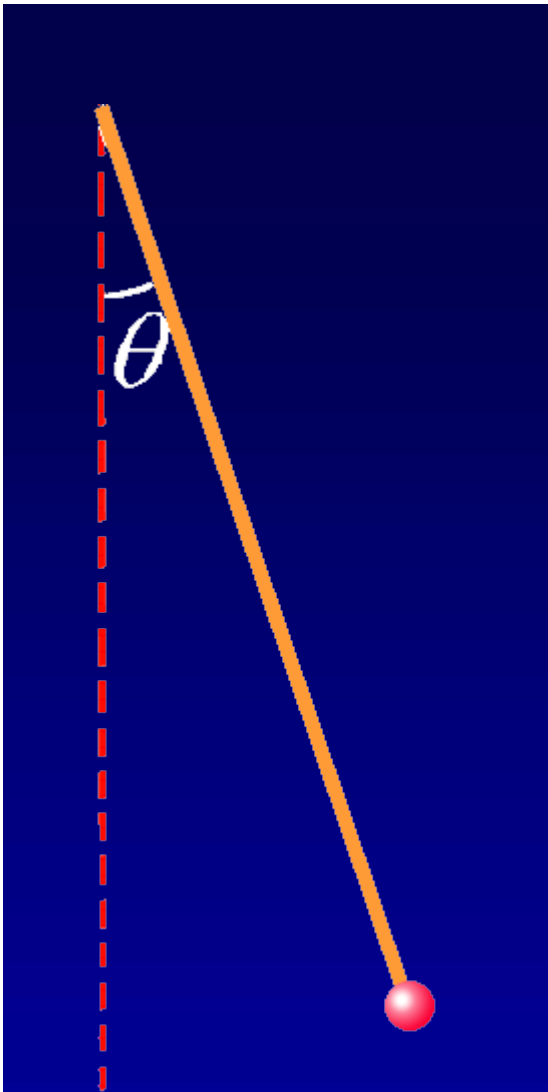
$$\theta = \theta_0 \cos \omega t$$

$$\dot{\theta} = -\omega \theta_0 \sin \omega t$$

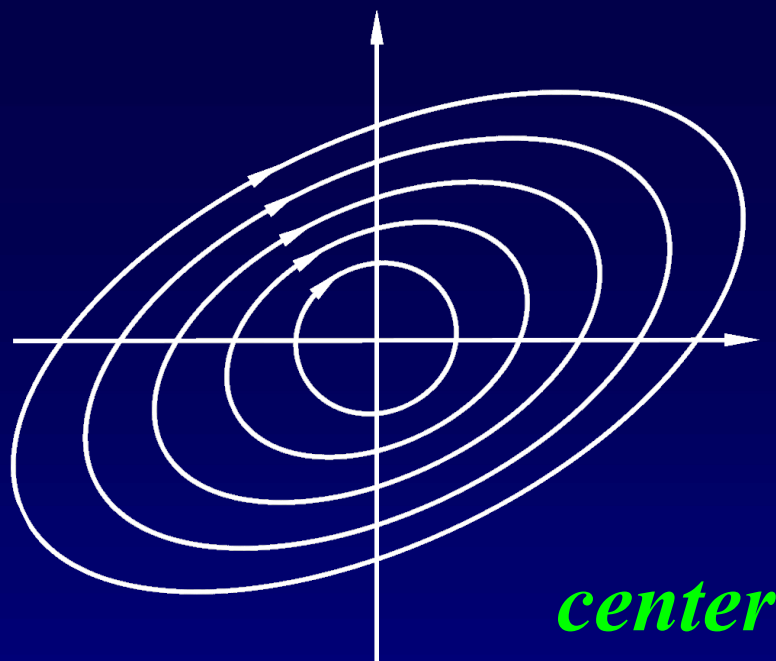
“trajectory”

$$\left(\frac{\theta}{\theta_0} \right)^2 + \left(\frac{\dot{\theta}}{\omega \theta_0} \right)^2 = 1$$

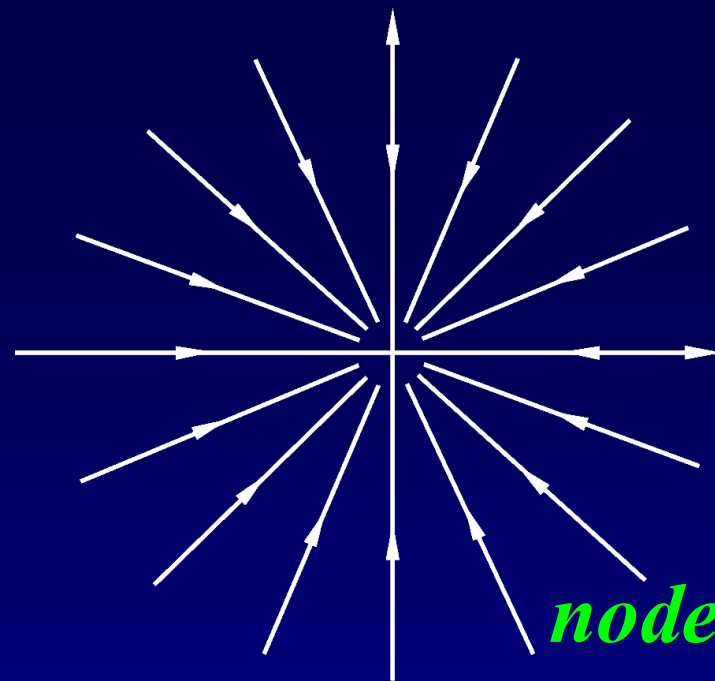




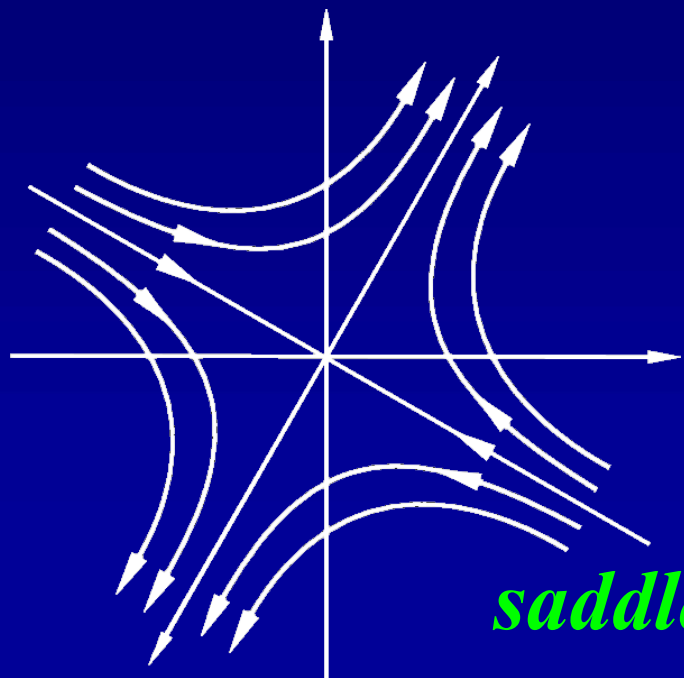
Complete kinematics
should include **phase diagram**.



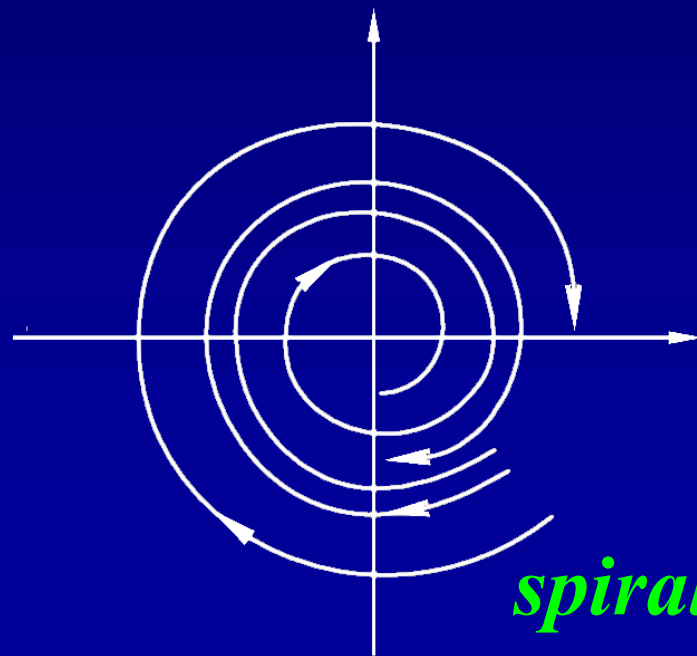
center



node



saddle



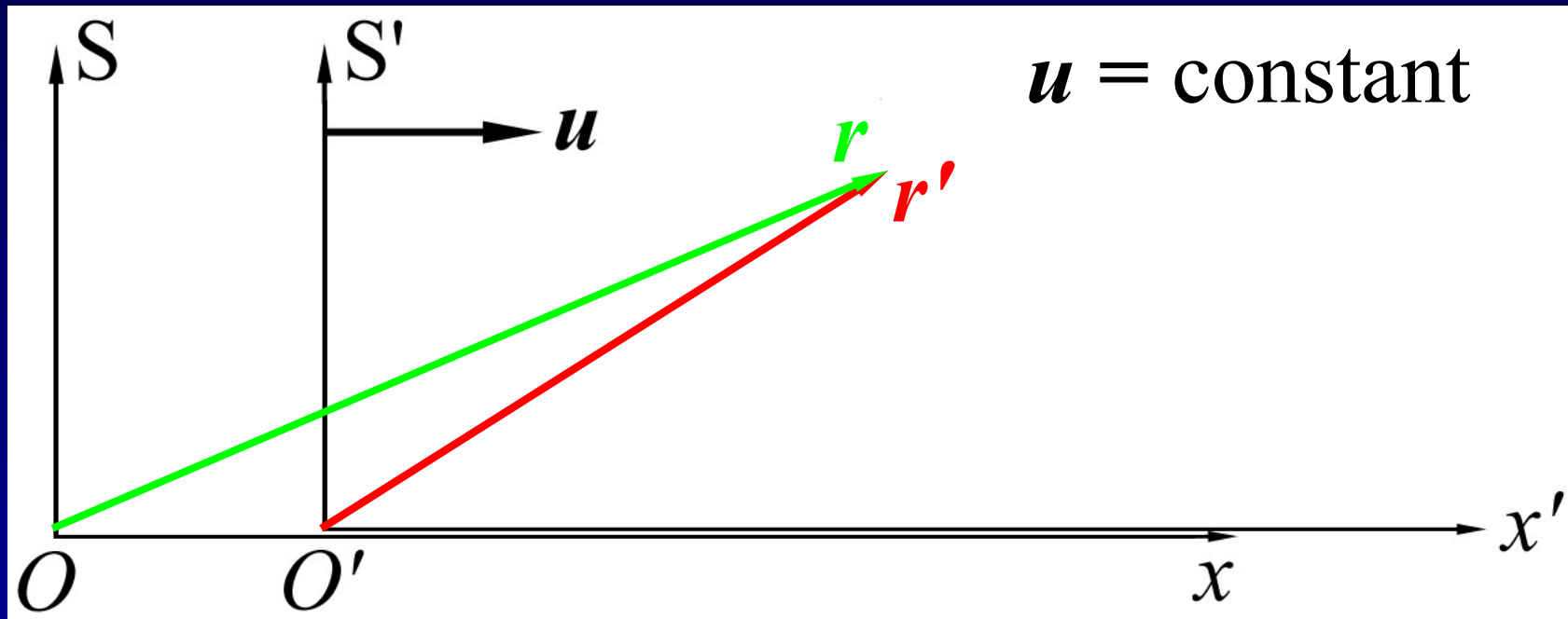
spiral

Strange attractors in phase space →

Chaotic motions

Fractal structure

2.6 Galilean transformation



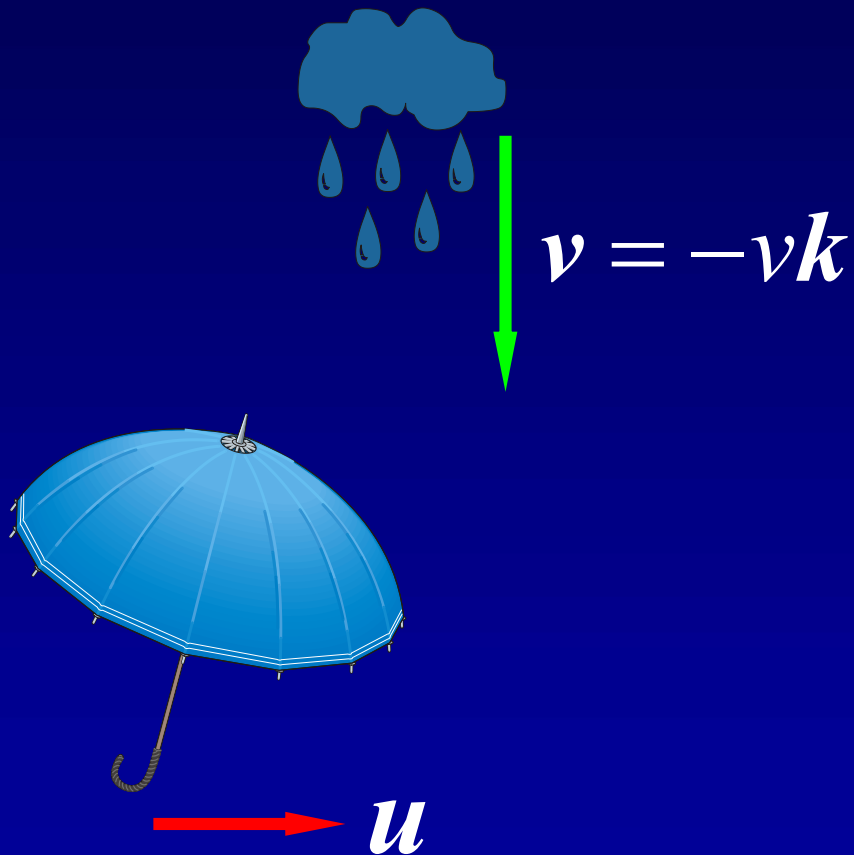
$$r' = r - \overrightarrow{OO'}$$

$$r' = r - u t$$

$$\dot{r}' = \dot{r} - u$$

$$\ddot{r}' = \ddot{r}$$

Walking in rain

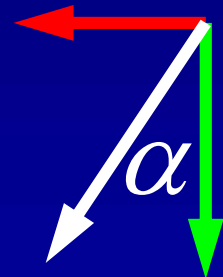


Solution:

Take walker as S' .

In walker's view,
the velocity of raindrop is:

$$\mathbf{v}' = \mathbf{v} - \mathbf{u} = -vk - ui$$



$$\tan \alpha = \frac{u}{v}$$

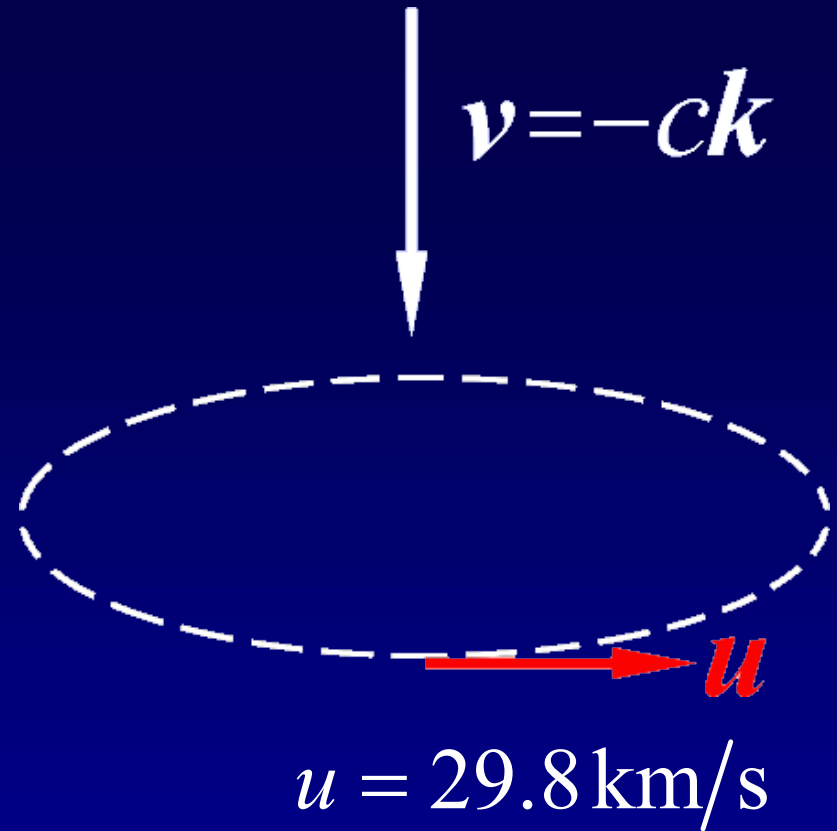
Aberration of light

光行差

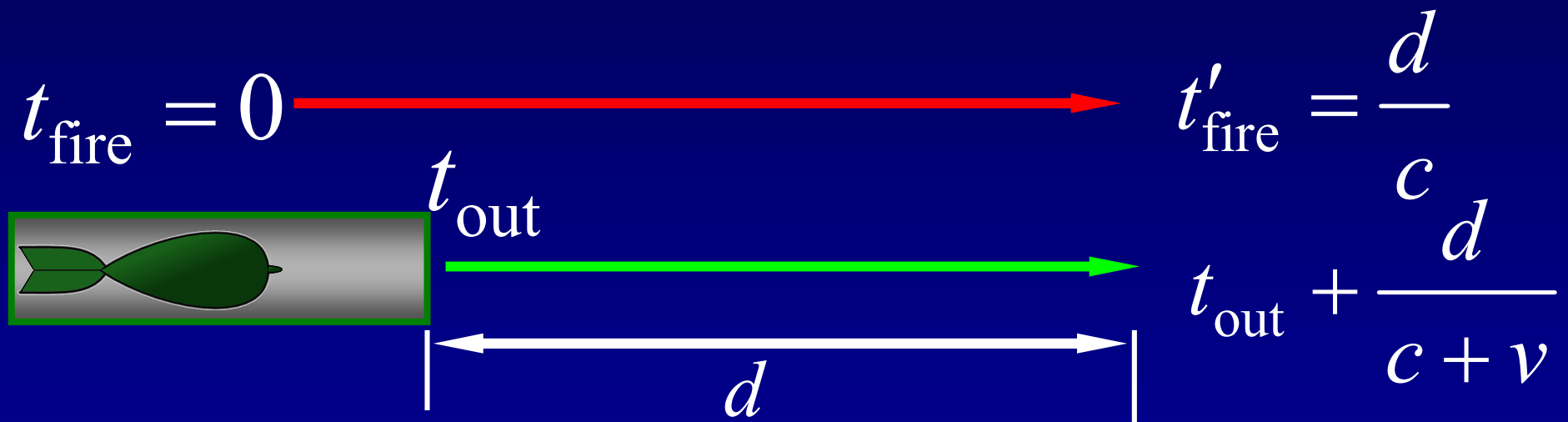
Solution:

$$\alpha = \arctan \frac{u}{c} \sim 10^{-4}$$

roughly correct



Gedenkanexperiment (thought experiment)



$$t'_{\text{f}} = \frac{d}{c} < t'_{\text{out}} = t_{\text{out}} + \frac{d}{c + v}$$

$$t_{\text{out}} > d \left(\frac{1}{c} - \frac{1}{c + v} \right)$$

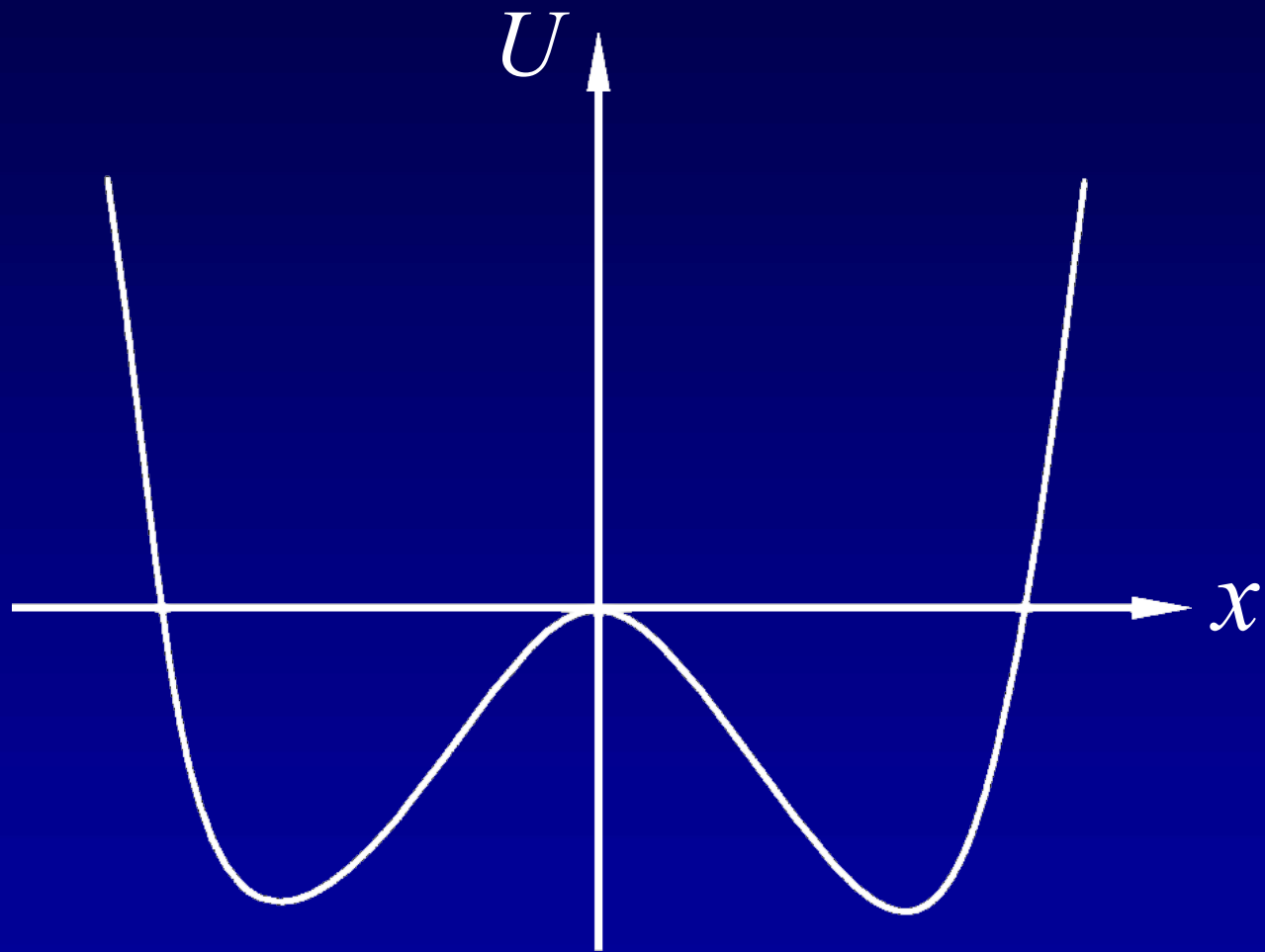
Large enough d may break the inequality.

So,

Galilean transformation is not valid for light.

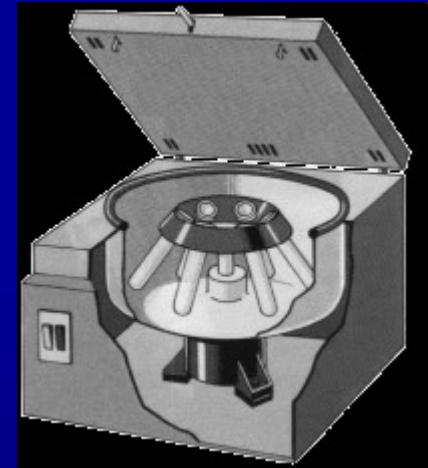
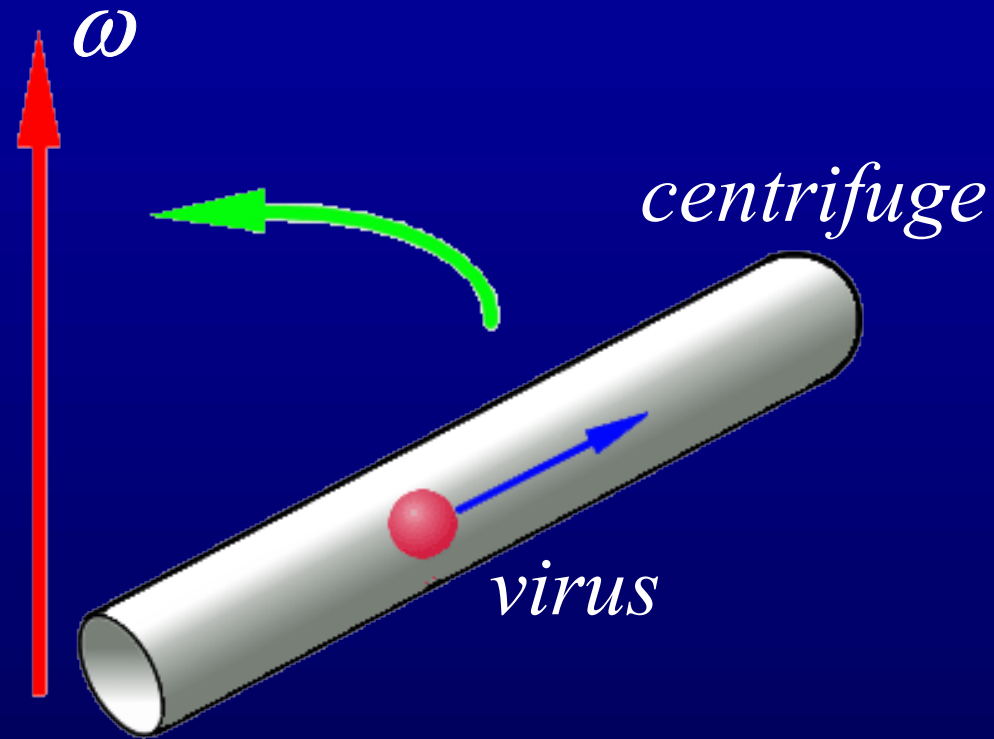
Fig.3.19&7.10

Assignment: 2.9, *2.13, 2.14, 2.15



Double-well potential

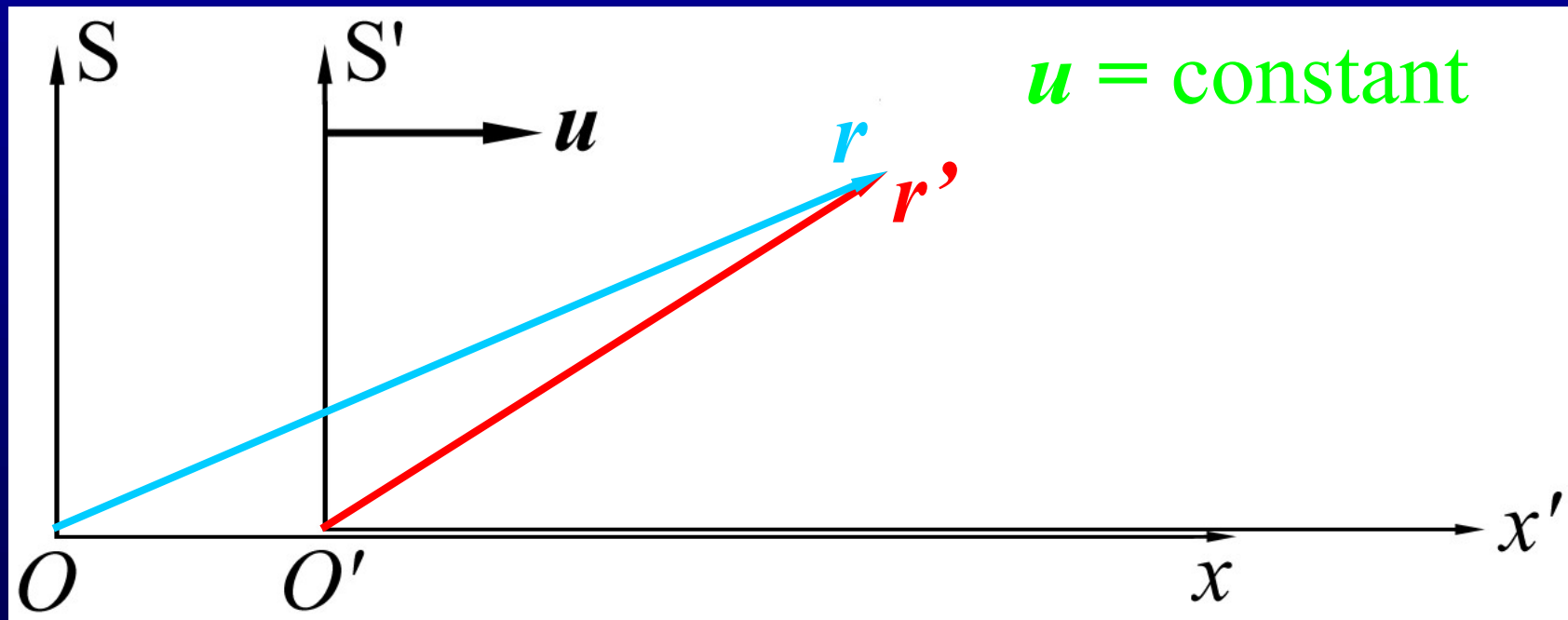
2.7 Coriolis acceleration



“What is the **trajectory** of the virus relative to the tube surface?” etc.

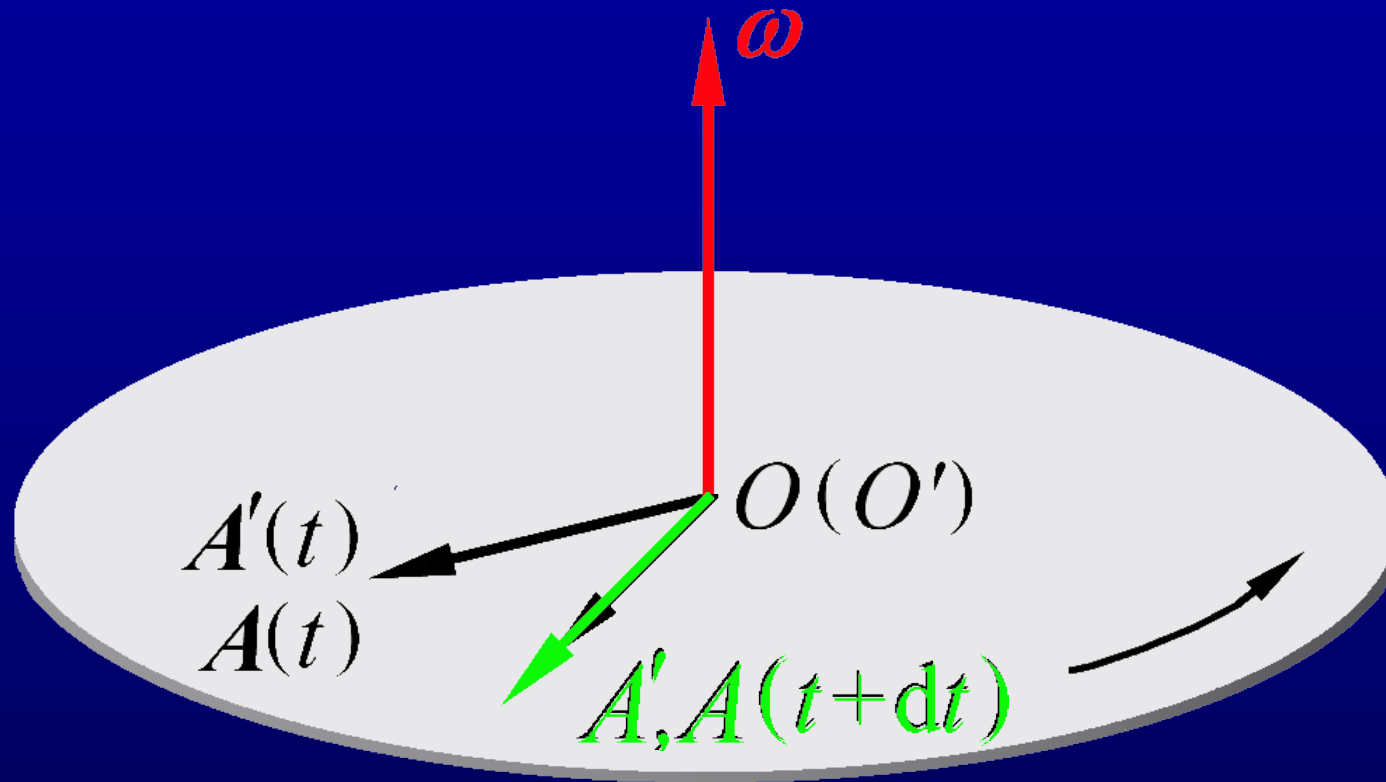
Transformation between coordinate systems

Between translational systems

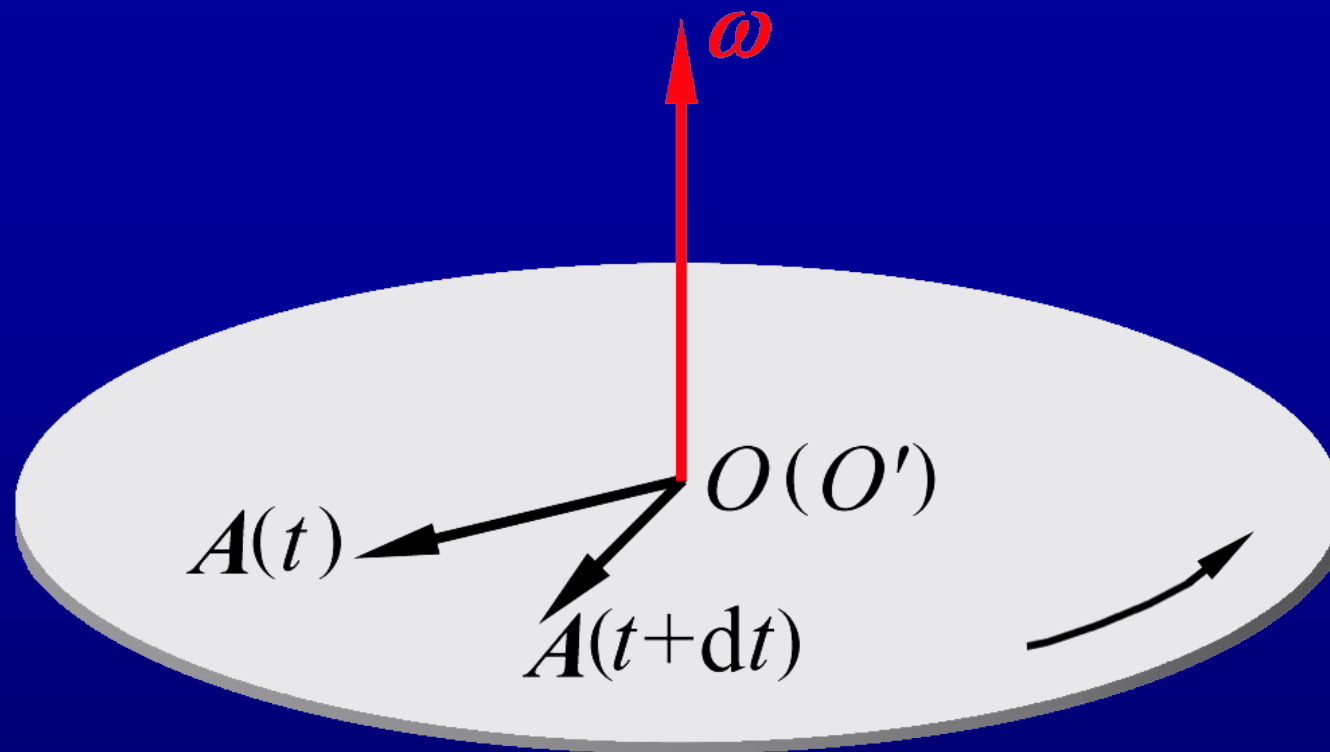


One particle, two systems, two position vectors

Between rotational systems



One particle, two systems, one position vectors,
two polar axes, two representations.



For A fixed in $\mathbf{S}'$,
 its end point is in CM relative to $\mathbf{S}$.

$$\begin{aligned} dA &= A(t+dt) - A(t) = (\omega \times A)dt \\ &\sim \dot{\rho} = \omega \times \rho \rightarrow d\rho = (\omega \times \rho)dt \end{aligned}$$

For additional dA' with respect to S'

$$dA = dA' + (\omega \times A)dt$$

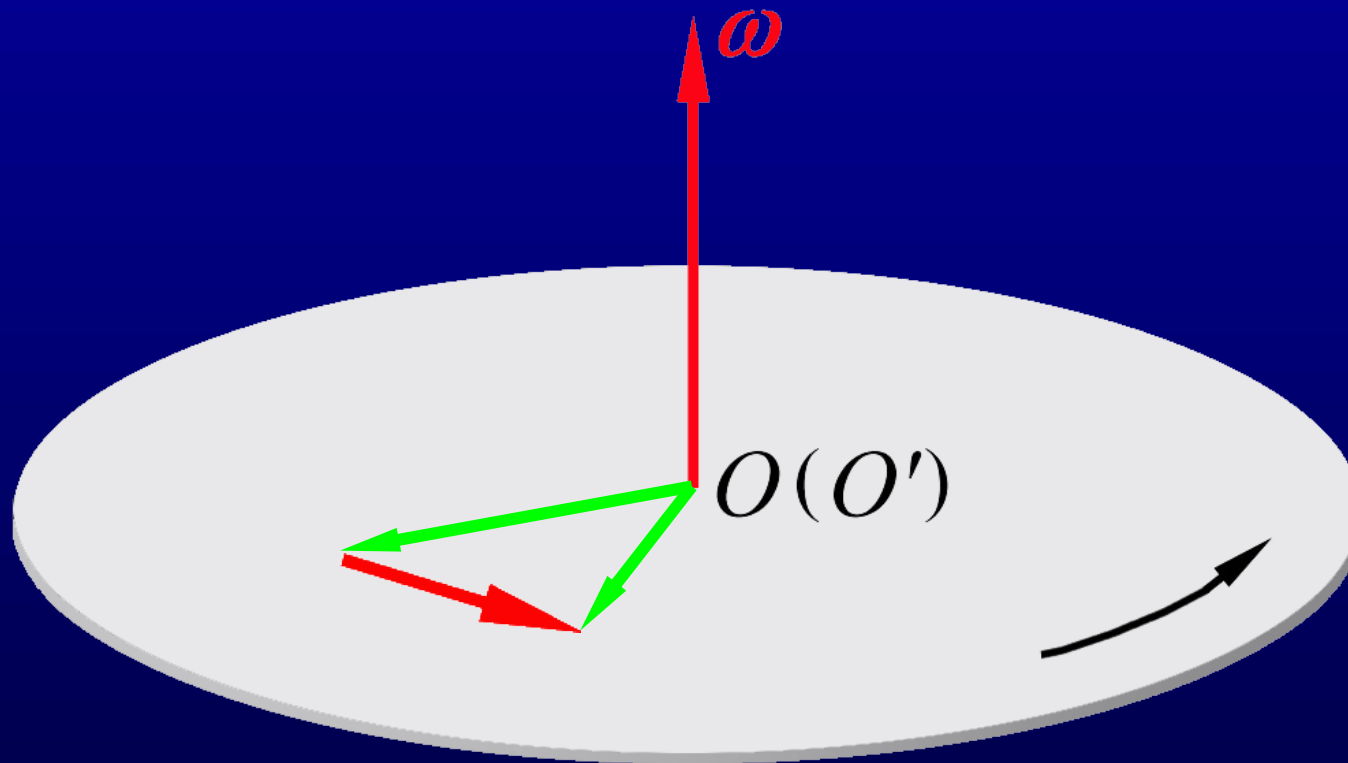
or

$$\left(\frac{dA}{dt} \right)_S = \left(\frac{dA}{dt} \right)_{S'} + \omega \times A$$

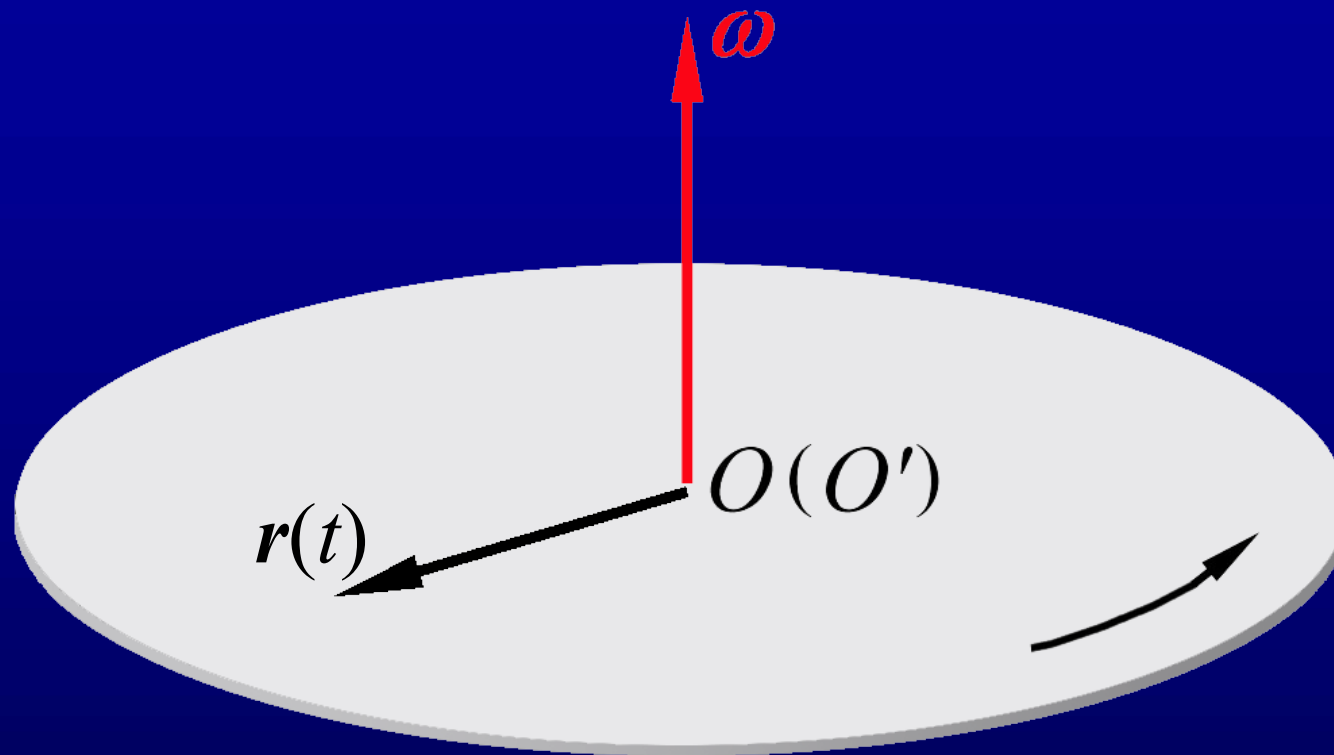
Symbolically

$$\frac{d}{dt} \boxed{} = \frac{d}{dt'} \boxed{} + \omega \times \boxed{}$$

For vectors not starting from the origin, an addition (only an addition) is needed.



For position vector r



$$\frac{d\mathbf{r}}{dt} = \frac{d\mathbf{r}}{dt'} + \boldsymbol{\omega} \times \mathbf{r}$$

$$\frac{d\dot{\mathbf{r}}}{dt} = \frac{d\dot{\mathbf{r}}}{dt'} + \boldsymbol{\omega} \times \dot{\mathbf{r}} \quad \boldsymbol{\omega} = \text{const.}$$

$$= \frac{d}{dt'} \left(\frac{d\mathbf{r}}{dt'} + \boldsymbol{\omega} \times \mathbf{r} \right) + \boldsymbol{\omega} \times \left(\frac{d\mathbf{r}}{dt'} + \boldsymbol{\omega} \times \mathbf{r} \right)$$

$$= \frac{d^2 \mathbf{r}}{dt'^2} + 2\boldsymbol{\omega} \times \frac{d\mathbf{r}}{dt'} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$$

Coriolis acceleration

centripetal acceleration

$$\mathbf{A} = \mathbf{r}, \quad \left(\frac{d\mathbf{r}}{dt} \right)_S = \left(\frac{d\mathbf{r}}{dt} \right)_{S'} + \boldsymbol{\omega} \times \mathbf{r}$$

$$\boxed{\left(\frac{d\mathbf{A}}{dt} \right)_S = \left(\frac{d\mathbf{A}}{dt} \right)_{S'} + \boldsymbol{\omega} \times \mathbf{A}}$$

$$\mathbf{A} = \left(\frac{d\mathbf{r}}{dt} \right)_S,$$

$$\left(\frac{d^2\mathbf{r}}{dt^2} \right)_S = \left[\frac{d}{dt} \left(\frac{d\mathbf{r}}{dt} \right)_S \right]_S = \left[\frac{d}{dt} \left(\frac{d\mathbf{r}}{dt} \right)_S \right]_{S'} + \boldsymbol{\omega} \times \left(\frac{d\mathbf{r}}{dt} \right)_S$$

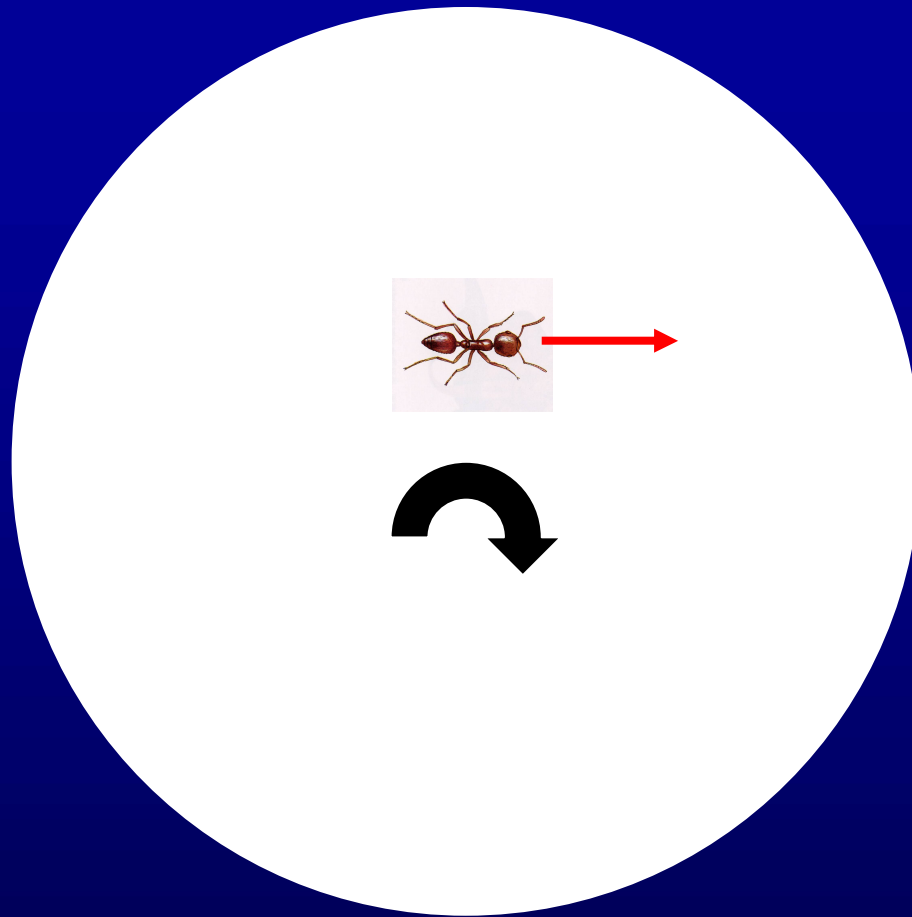
$$= \left\{ \frac{d}{dt} \left[\left(\frac{d\mathbf{r}}{dt} \right)_{S'} + \boldsymbol{\omega} \times \mathbf{r} \right] \right\}_{S'} + \boldsymbol{\omega} \times \left[\left(\frac{d\mathbf{r}}{dt} \right)_{S'} + \boldsymbol{\omega} \times \mathbf{r} \right]$$

$$= \left[\frac{d}{dt} \left(\frac{d\mathbf{r}}{dt} \right)_{S'} \right]_{S'} + \boldsymbol{\omega} \times \left(\frac{d\mathbf{r}}{dt} \right)_{S'} + \boldsymbol{\omega} \times \left(\frac{d\mathbf{r}}{dt} \right)_{S'} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$$

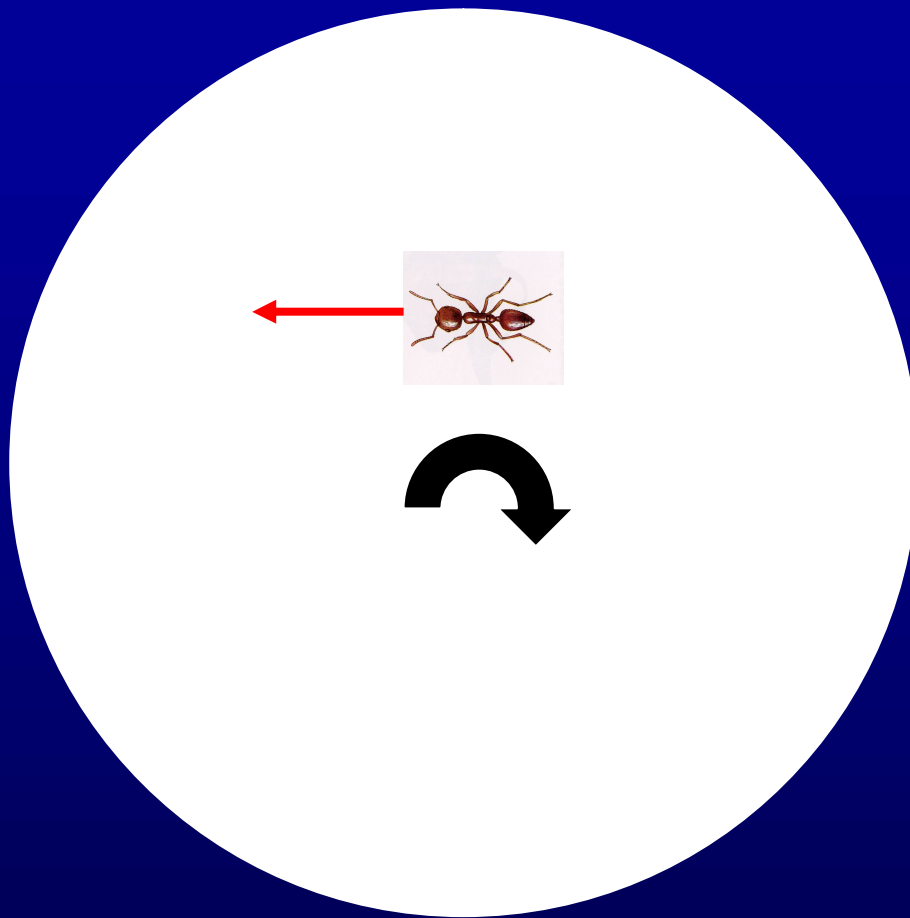
$$= \left[\frac{d}{dt} \left(\frac{d\mathbf{r}}{dt} \right)_{S'} \right]_{S'} + \underline{2\boldsymbol{\omega} \times \left(\frac{d\mathbf{r}}{dt} \right)_{S'}} + \underline{\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})}$$

Coriolis acceleration

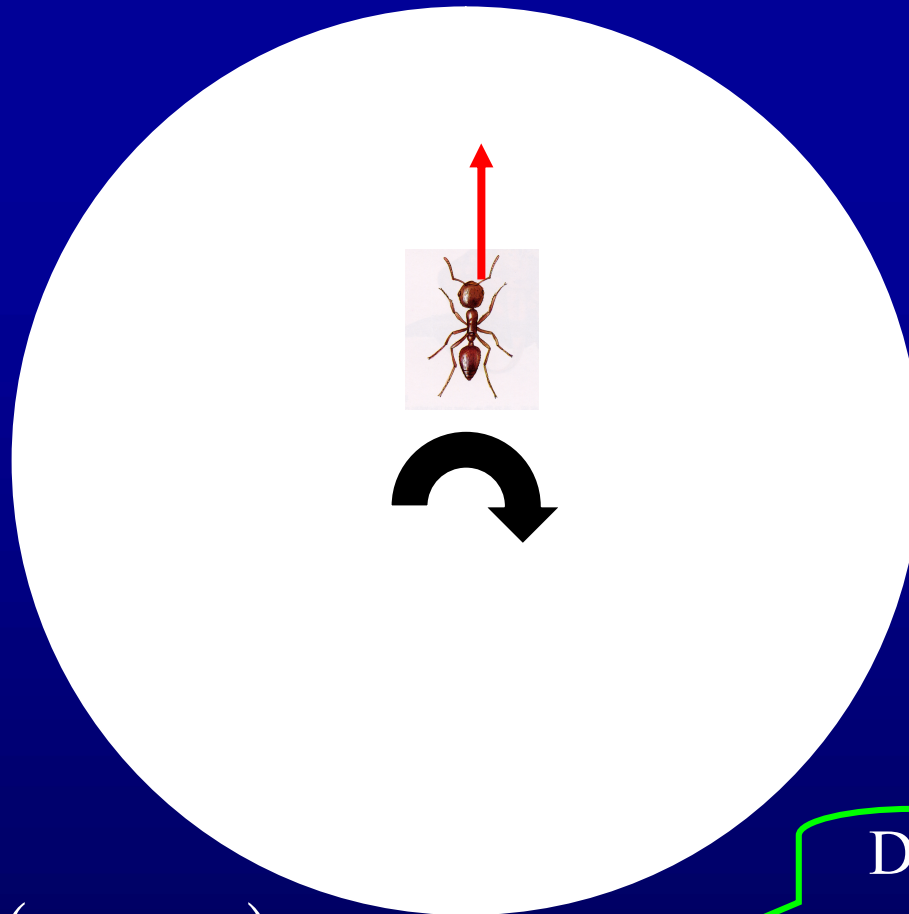
centripetal acceleration



$$\Delta a = \frac{(\omega r + v)^2}{r} - \frac{\omega^2 r^2}{r} \approx \frac{2\omega v r}{r} = 2\omega v$$



$$\Delta a = \frac{(\omega r - v)^2}{r} - \frac{\omega^2 r^2}{r} \approx -\frac{2\omega v r}{r} = -2\omega v$$



$$\Delta v = \omega \Delta t v$$



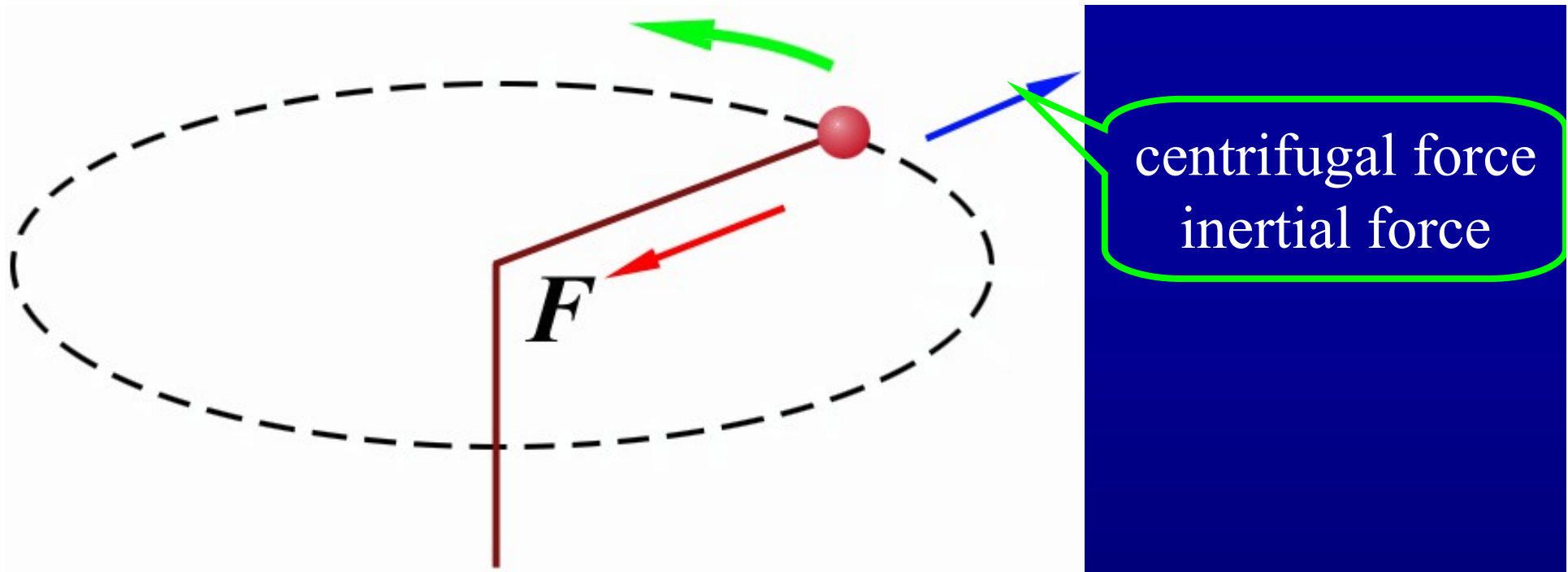
$$(\Delta a)_1 = \frac{\omega(r + v\Delta t) - \omega r}{\Delta t} = \omega v$$

Due to the change of
the position

$$(\Delta a)_2 = \frac{\omega \Delta t v}{\Delta t} = \omega v.$$

Due to the change of
the direction of the velocity

$$\Delta a = (\Delta a)_1 + (\Delta a)_2 = 2\omega v$$



In S , F is centripetal force which causes centripetal acceleration a_c .
 S frame: $F = ma_c$.

In S' , the ball is at rest, no dr/dt' , d^2r/dt'^2 .

non-inertial frame

S' frame: $F + F_{in} = F - ma_c = 0$.

$$m \left(\frac{d^2 \mathbf{r}}{dt^2} \right)_S = m [\mathbf{g} + 2\boldsymbol{\omega} \times \mathbf{v}' + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})] = m \mathbf{g}_0$$

gravity

$$\mathbf{g} = \left(\frac{d^2 \mathbf{r}}{dt^2} \right)_{S'}$$

In the S' frame that rotates with the earth

$$m \mathbf{g} = m \mathbf{g}_0 - 2m \boldsymbol{\omega} \times \mathbf{v}' - m \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$$

In S'

Coriolis "force"

Centrifugal 'force'

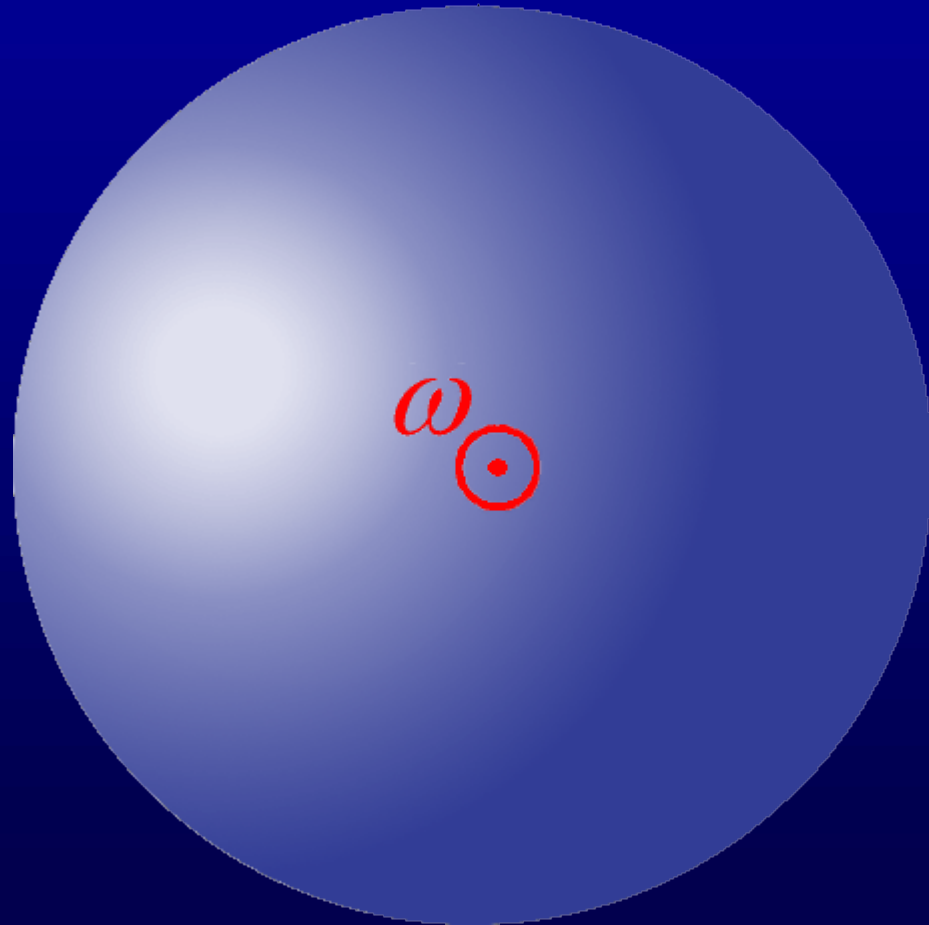
$$g_0 \approx 9.8 \text{ m/s}^2$$

$$\omega = 7.292 \times 10^{-5} \text{ rad/s}$$

$$|\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})| \leq \omega^2 R \approx 3.39 \times 10^{-2} \text{ m/s}^2 \ll g_0$$

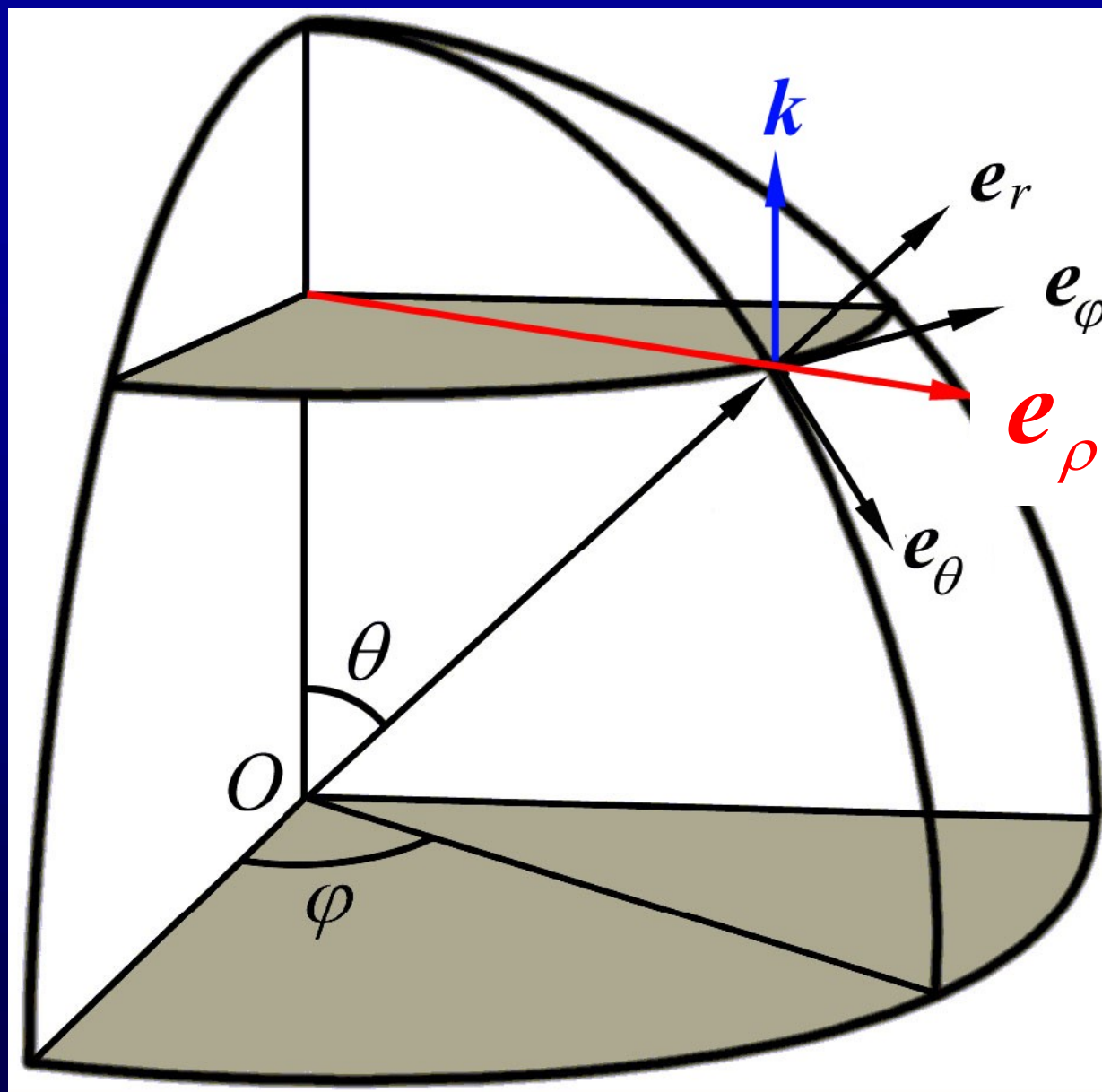
$$m \mathbf{g} = m \mathbf{g}_{\text{eff}} - 2m \boldsymbol{\omega} \times \mathbf{v}', \quad \mathbf{g}_{\text{eff}} = \mathbf{g}_0 - \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$$

falling body



$$\begin{aligned}
-2m\omega \times \mathbf{v}' &\approx -2m\omega \mathbf{k} \times \mathbf{v}'(-\mathbf{e}_r) \\
&= 2m\omega v'(\mathbf{k} \times \mathbf{e}_r) \\
&= 2m\omega v' \sin \theta \mathbf{e}_\varphi
\end{aligned}$$

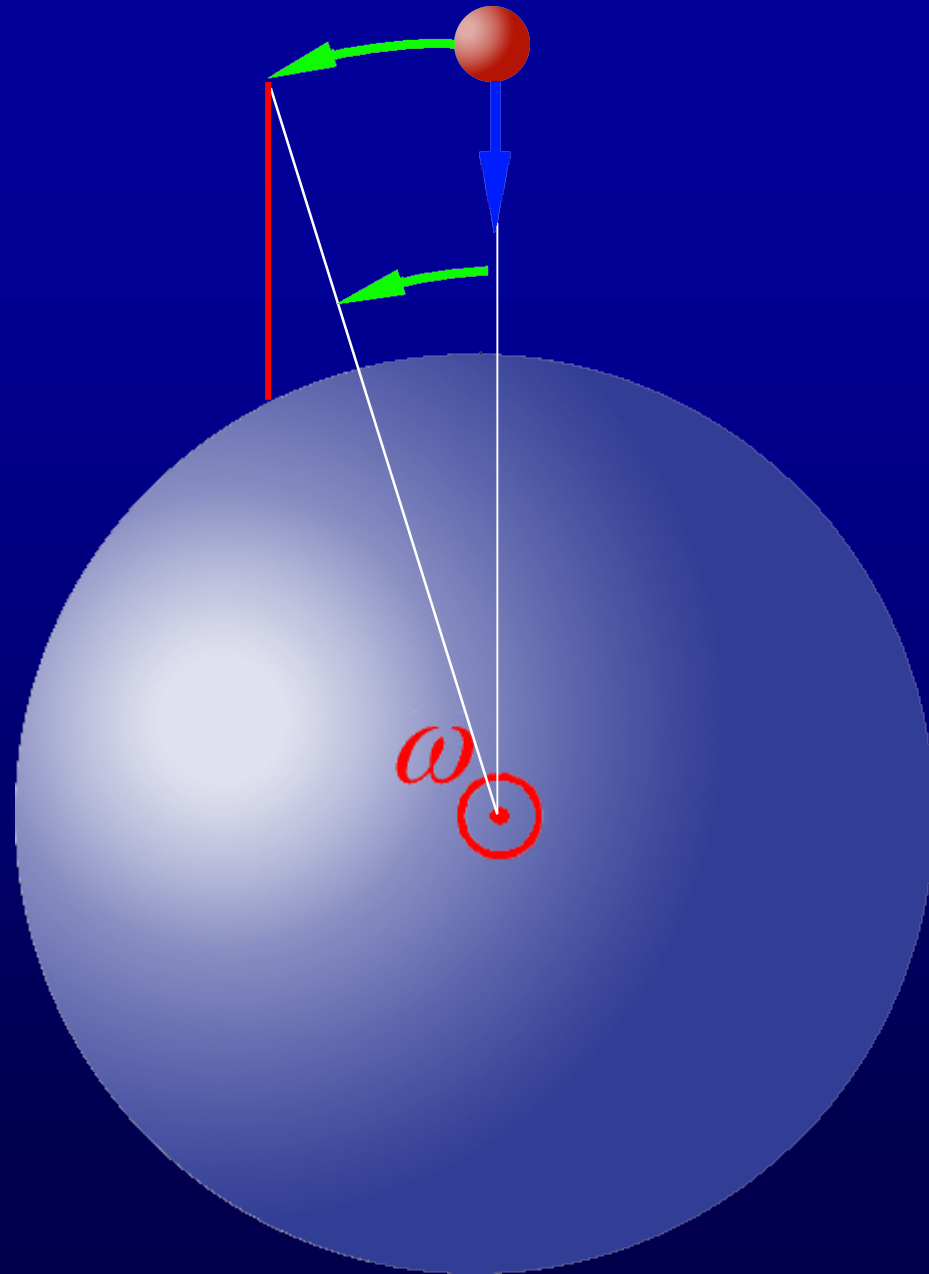
$$\begin{aligned}
\mathbf{k} \times \mathbf{e}_r &= \mathbf{k} \times (\cos \theta \mathbf{k} + \sin \theta \mathbf{e}_\rho) \\
&= \sin \theta \mathbf{e}_\varphi
\end{aligned}$$



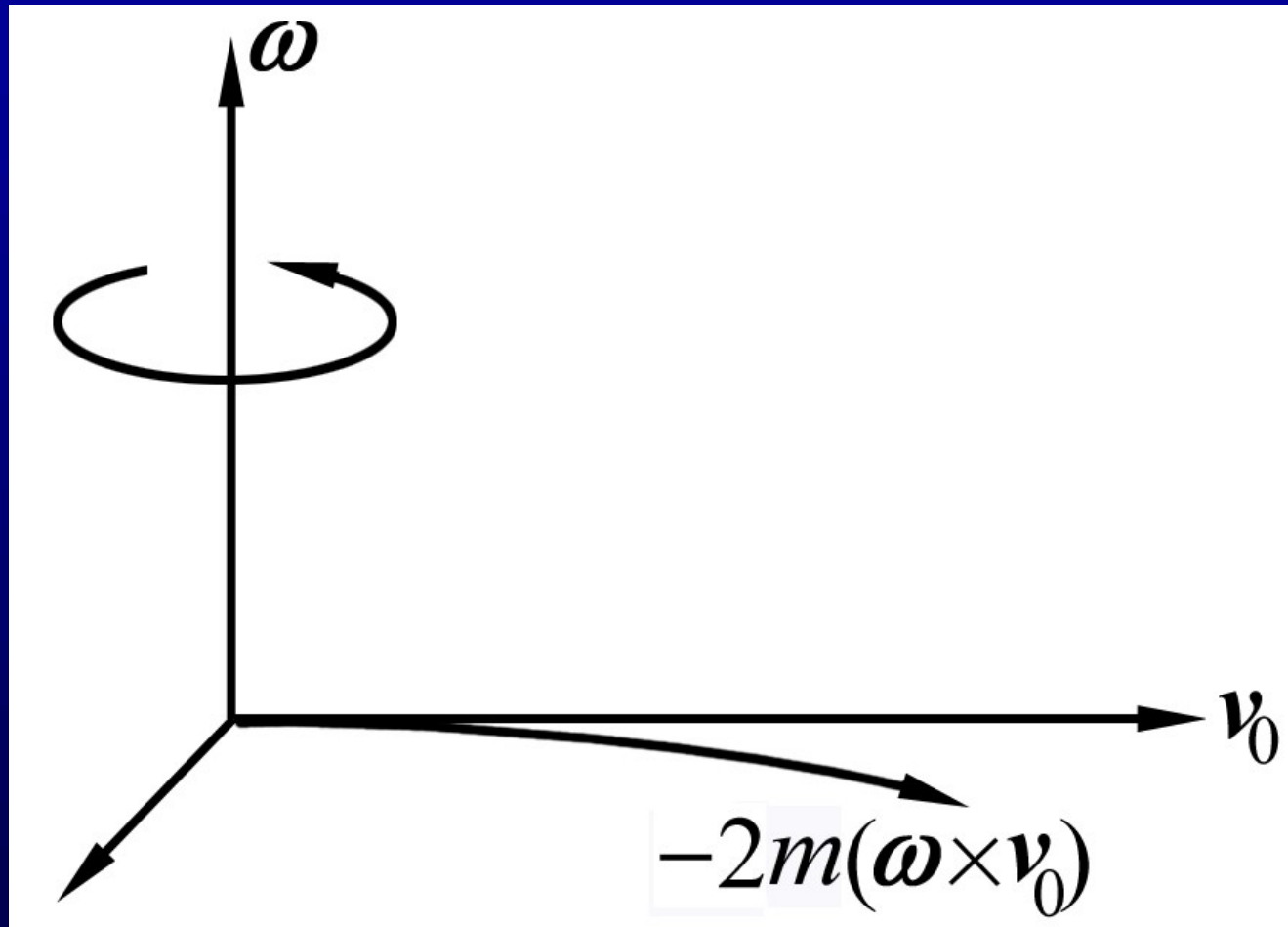
$$\begin{aligned}
-2m\omega \times \mathbf{v}' &\approx -2m\omega \mathbf{k} \times v'(-\mathbf{e}_r) \\
&= 2m\omega v'(\mathbf{k} \times \mathbf{e}_r) \\
&= 2m\omega v' \sin \theta \mathbf{e}_\varphi \\
&\text{eastward if } \theta \neq 0
\end{aligned}$$

$$\begin{aligned}
\mathbf{k} \times \mathbf{e}_r &= \mathbf{k} \times (\cos \theta \mathbf{k} + \sin \theta \mathbf{e}_\rho) \\
&= \sin \theta \mathbf{e}_\varphi
\end{aligned}$$

falling body
Kinematic effect



On the surface



On the surface $\mathbf{v} = v_\theta \mathbf{e}_\theta + v_\phi \mathbf{e}_\phi$

$$\begin{aligned} -2\boldsymbol{\omega} \times (v_\theta \mathbf{e}_\theta + v_\phi \mathbf{e}_\phi) &= 2\boldsymbol{\omega} (v_\theta \mathbf{e}_\theta \times \mathbf{k} + v_\phi \mathbf{e}_\phi \times \mathbf{k}) \\ &= 2\boldsymbol{\omega} (-v_\theta \cos \theta \mathbf{e}_\phi + v_\phi \mathbf{e}_\rho) \end{aligned}$$

On the surface $\mathbf{v} = v_\theta \mathbf{e}_\theta + v_\phi \mathbf{e}_\phi$

$$\begin{aligned} -2\omega \times (v_\theta \mathbf{e}_\theta + v_\phi \mathbf{e}_\phi) &= 2\omega (v_\theta \mathbf{e}_\theta \times \mathbf{k} + v_\phi \mathbf{e}_\phi \times \mathbf{k}) \\ &= 2\omega (-v_\theta \cos \theta \mathbf{e}_\phi + v_\phi \mathbf{e}_\rho) \\ &= 2\omega \cos \theta (-v_\theta \mathbf{e}_\phi + v_\phi \mathbf{e}_\theta) + 2\omega v_\phi \sin \theta \mathbf{e}_r \end{aligned}$$

$v_\theta = 0$; $v_\phi \mathbf{e}_\phi \rightarrow v_\phi \mathbf{e}_\theta$ east $\rightarrow$ south

righthand?

$v_\phi = 0$; $v_\theta \mathbf{e}_\theta \rightarrow -v_\theta \mathbf{e}_\phi$ south $\rightarrow$ west

In southern hemisphere $\cos \theta < 0$

Coriolis effect

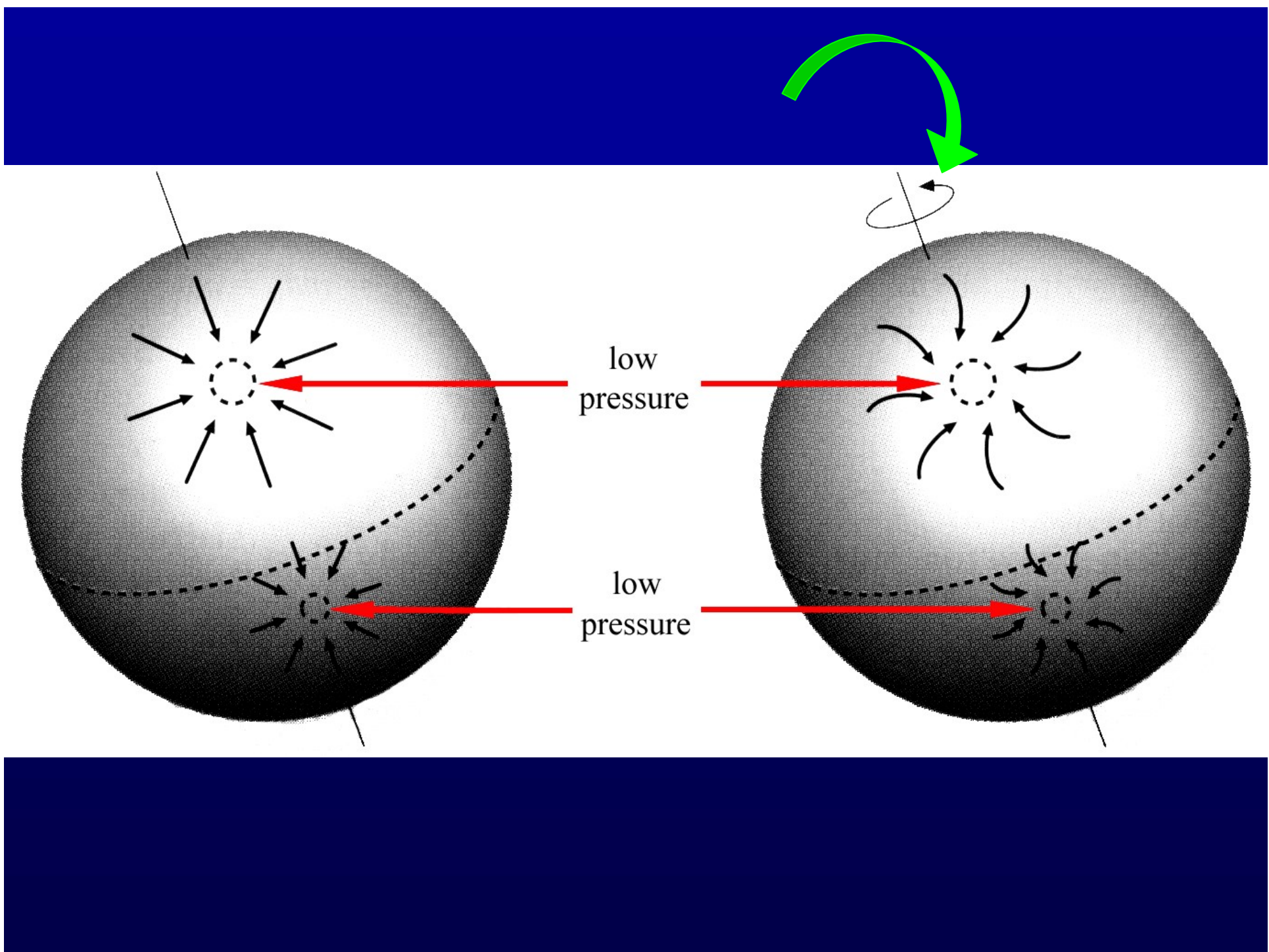
Railway tracks

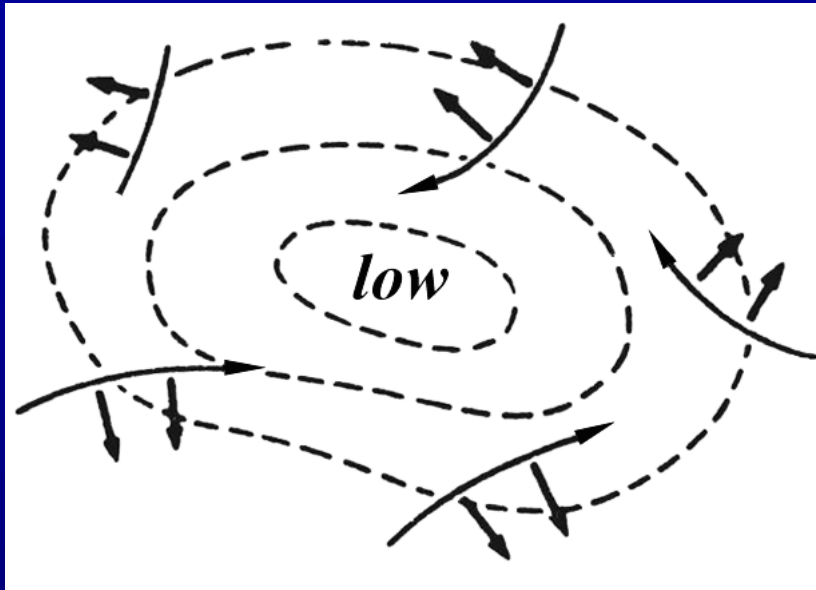
river-bank

Molecular rotation-oscillation spectrum

Whirling in satellite cloud diagram.

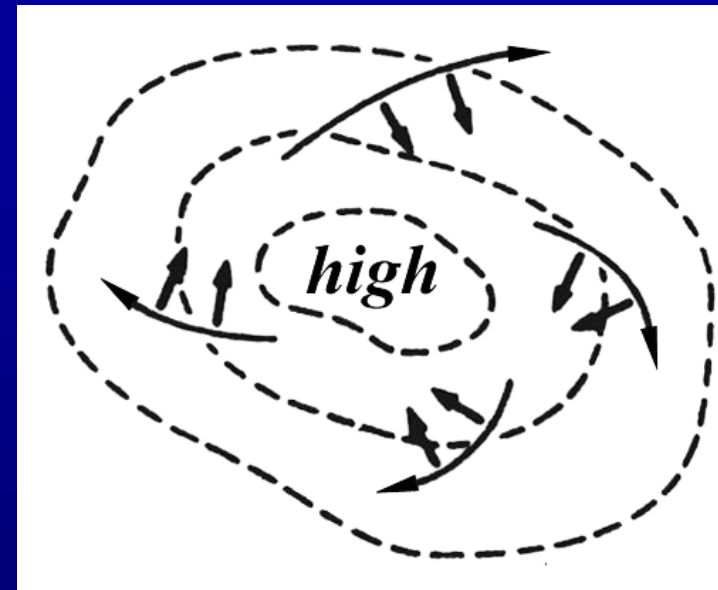
**It is counterclockwise in northern hemisphere,
and clockwise in southern hemisphere.**





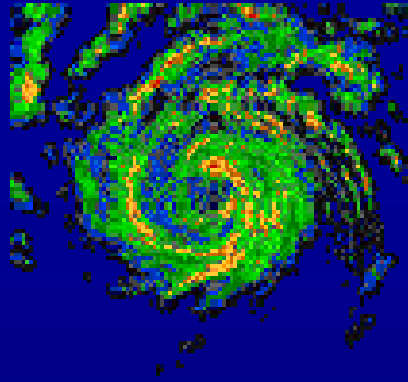
A low pressure system (cyclone) develops where relatively warm air ascends from the Earth's surface.

Counterclockwise in North Hemisphere



A high pressure system (anticyclone) develops where relatively cold air sinks down from aloft.

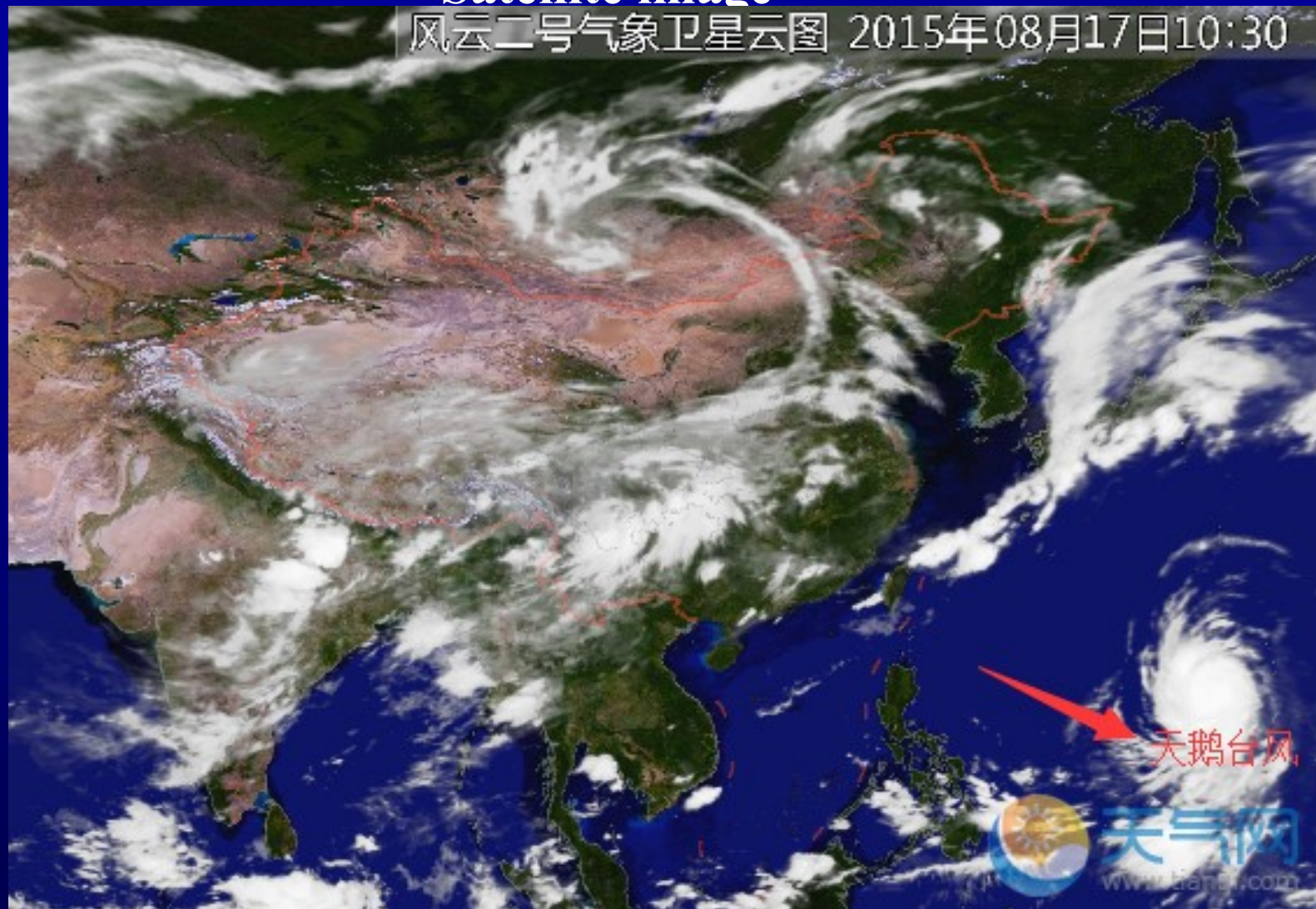
Clockwise in North Hemisphere



A tropical cyclone in the northern hemisphere

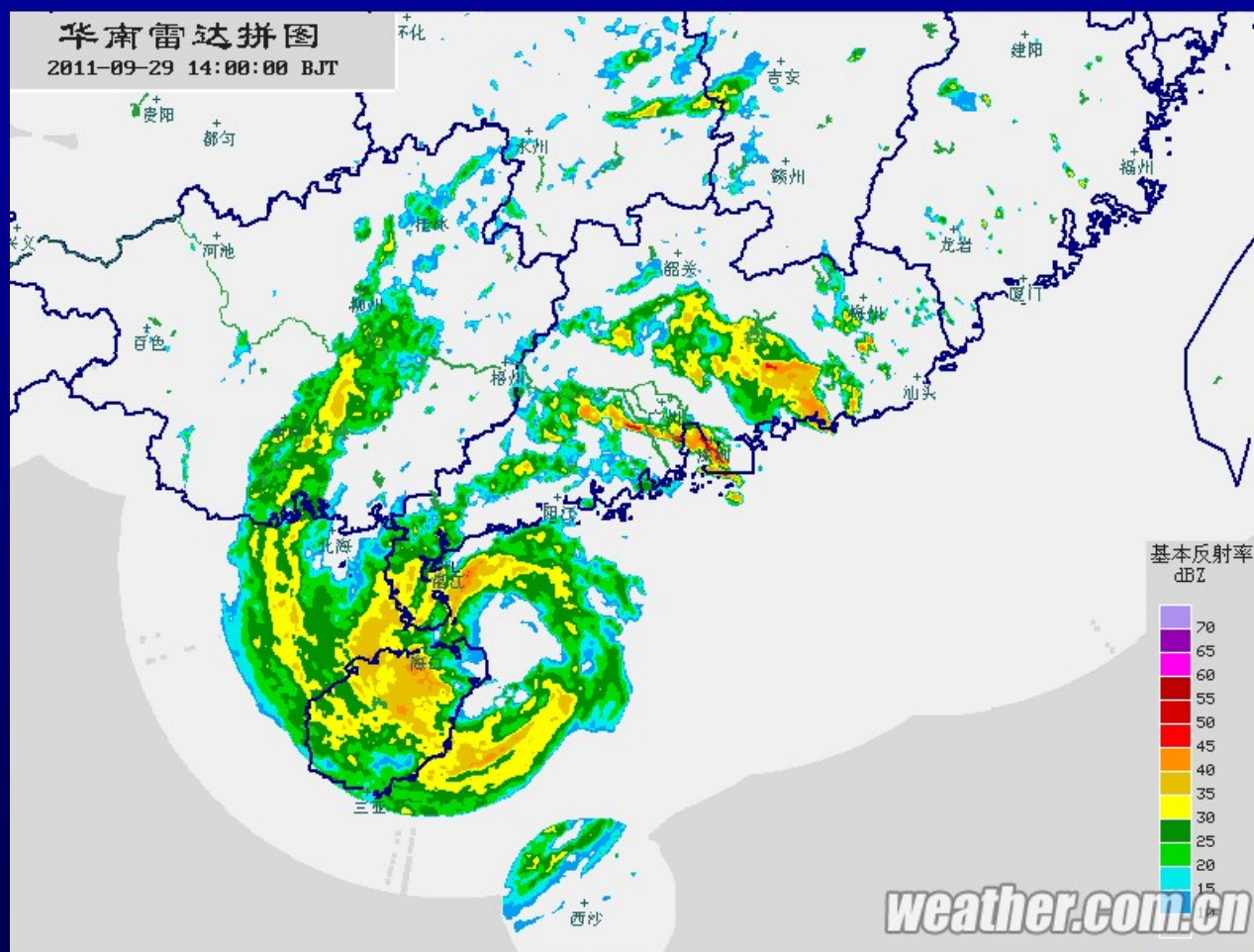
tropical cyclone (tropical depression, tropical storm, typhoon, or hurricane depending on strength and geographical context)

Satellite image



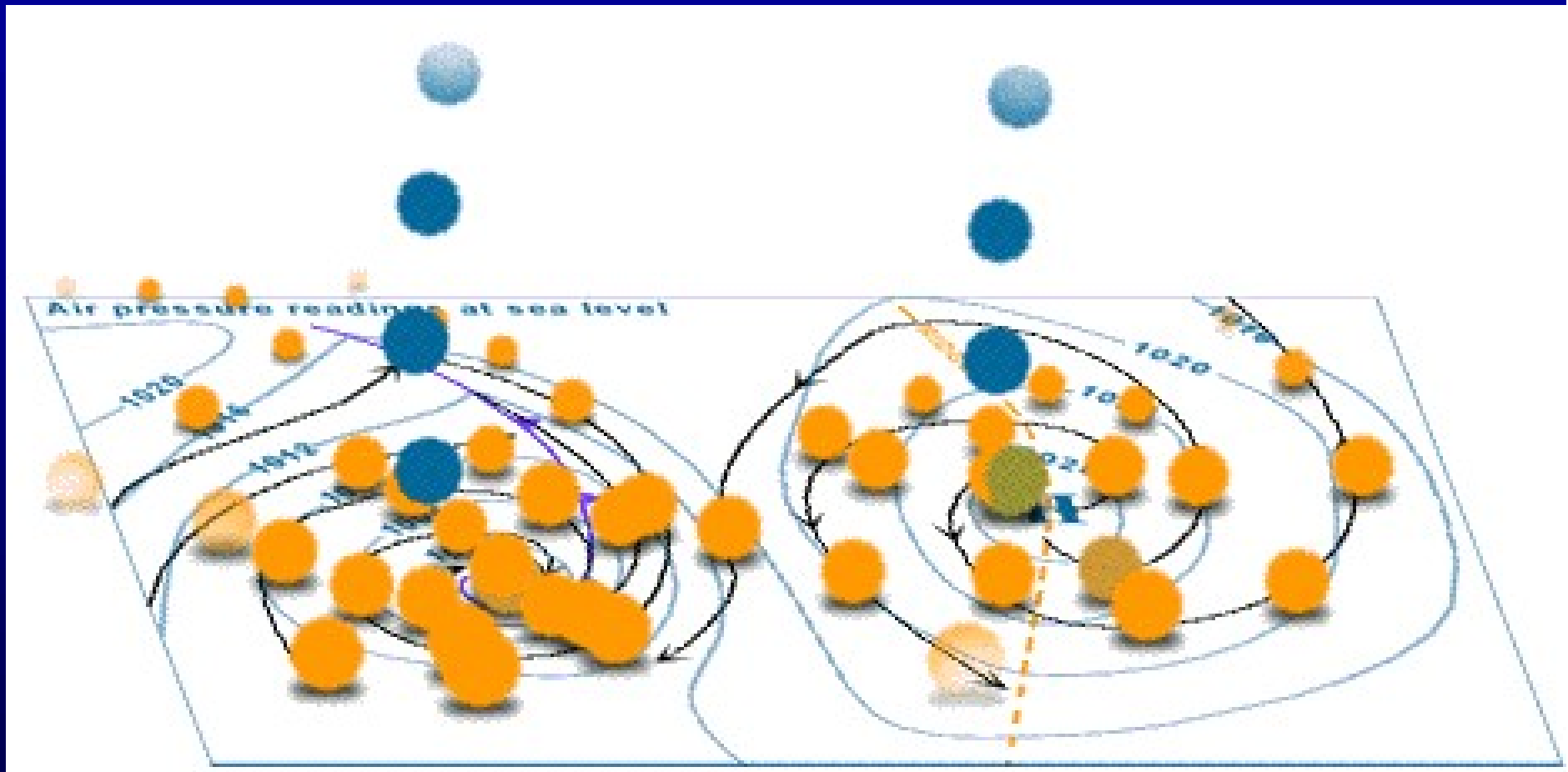
Typhoon Swan 天鹅

Weather radar image



Typhoon Nesat 纳沙

In Australia (South Hemisphere)

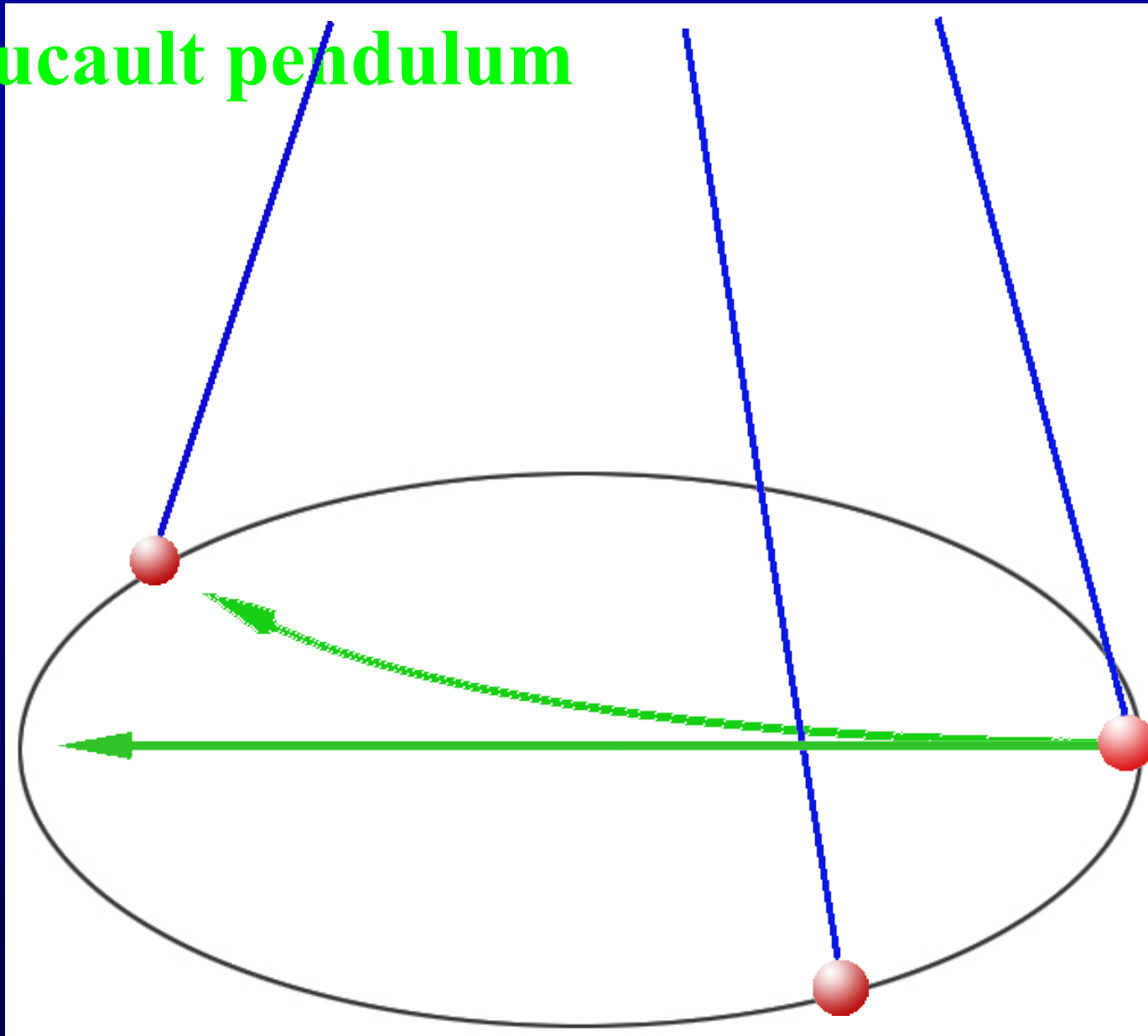


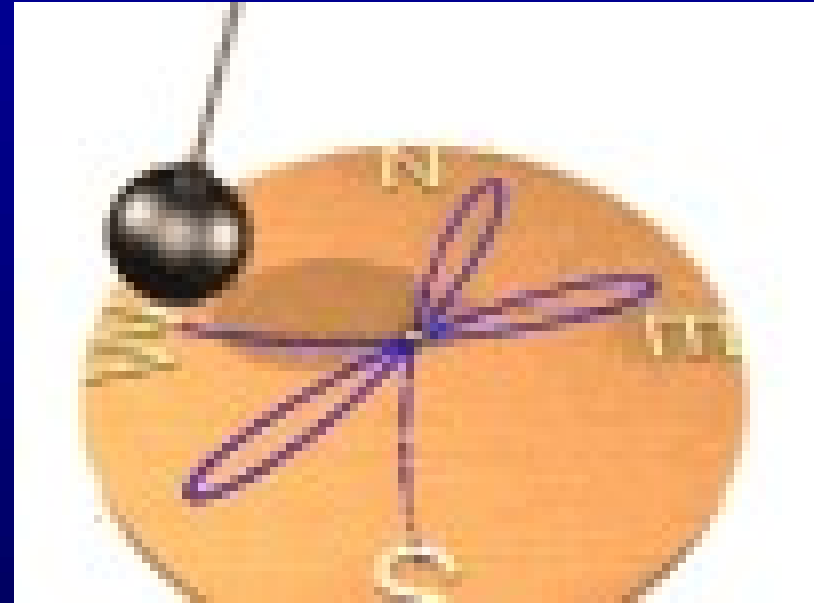
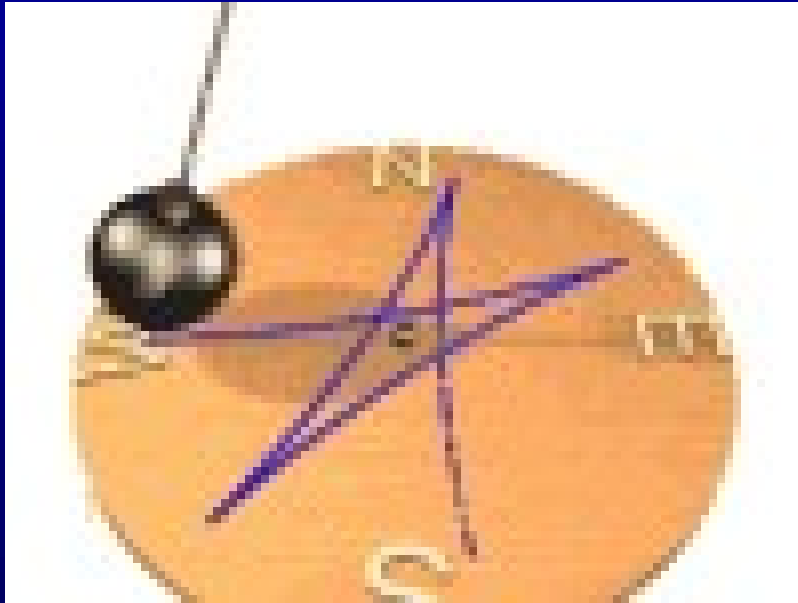


**Paris –
Pantheon (万神殿)
Foucault's pendulum**

**In 1851, Léon Foucault
made his most famous
pendulum when he
suspended a 28-kg bob
with a 67-metre wire
from the dome of the
Panthéon in Paris.**

Foucault pendulum





Assignment 2.16